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The "Lack" of Volatility Trade-Offs in Exchange Rate Zones with Sticky Prices

Elias D. Belessakos
New York Life Investment Management, U.S.A.

Christos I. Giannikos*
Department of Economics and Finance, Baruch College, U.S.A.

Abstract

In target zone regimes, volatility trade-offs between the nominal exchange rate and the nominal interest rate differential depend on the underlying monetary model assumption. In an economy with price rigidities there exists no such trade-off when the exchange rate overshoots.

Key words: exchange rate; target zones; volatility trade-offs; sticky prices; overshooting

JEL classification: F31; E43

1. Introduction

This paper deals with the relationship between the volatility of the nominal exchange rate and the volatility of the interest rate differential as well as the linkage between the volatility of exchange rate and that of output within the framework of sticky prices.

Svensson (1991a) studied the asymptotic (unconditional) variability of the exchange rate and the interest rate differential as well as the instantaneous (conditional) variability of the two variables. The model he uses is one of a perfectly credible target zone regime with flexible prices. He finds that there is a linear trade-off between the variability of the exchange rate and that of the nominal interest rate differential. Thus, a decrease in the volatility (conditional standard deviation) of the exchange rate leads to an increase in the volatility of the nominal interest rate differential. Svensson (1991b) used Swedish data to verify some of the predictions of this model.

However, data for European Monetary System (EMS) currencies indicate that a

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*Correspondence to: Department of Economics and Finance, Baruch College, One Bernard Baruch Way, Box: B10-225, New York, NY 10010, USA. E-mail:Christos_Giannikos@baruch.cuny.edu. We thank Harris Dellas, Cliff J. Huang, Deniz Ozenbas, Jong-Shin Wei, and, especially, two anonymous referees for helpful comments while we retain all responsibility for any remaining errors.

positive relationship prevails between the two variables. As Bertola and Caballero (1992) demonstrated, the positive relationship captured by the data for EMS currencies can, possibly, be due to realignment risk being related to the positions of EMS exchange rates within their bands. Separately, Lindberg and Soderlind (1991), working with an updated and longer set of Swedish data, also find a positive relationship between the exchange rate and the interest rate differential.

Indeed, when the commitment to the target zone is not perfectly credible, a trade-off of the type suggested by Svensson (1991a) might not exist. Bertola and Svensson (1993), in a first target zone model with time-varying realignment risk, show that the above negative relation prevails if the realignment risk is zero but, in the presence of non-zero realignment risk, the relationship between the volatility of the exchange rate and that of the interest rate differential can be negative or positive.

Clearly, the monetary model does not allow either for deviations of output from full employment or for purchasing power violations. In addition this model has the undesirable property that the fundamental is assumed to follow a Brownian motion with drift. Such a stochastic process is non-stationary with infinite asymptotic mean and variance. All shocks in the economy are permanent, and the fundamental does not exhibit any tendency to return to equilibrium. In one of the natural extensions of the basic target zone model intra-marginal intervention is allowed. Lindberg and Soderlind (1992) argue that this can be modeled as a mean-reverting process for fundamentals. Also an empirical puzzle documented by Beetsma and van der Ploeg (1998) (that exchange rate distributions are hump-shaped rather than U-shaped as predicted by the standard target zone model) for EMS exchange rates might be explained away in the presence of price sluggishness. Finally, Rogoff (1993), in a comment to Svensson (1993), argues for the potential benefits of modeling price rigidities as well.

In this paper, following the above leads in the literature, we use a stochastic version of the Dornbusch (1976) overshooting model as modified by Miller and Weller (1991) and later further explored by Sutherland (1994). In this model the process for the fundamental is endogenous since its drift depends on the exchange rate itself. Prices are sticky and, as a result, output does not remain always in equilibrium while the exchange rate does not satisfy the purchasing power parity at all times. We show that in an economy with sticky prices, and still under perfectly credible exchange rate bands, there exists no volatility trade-off between the exchange rate and the interest rate differential (there is a positive linkage between the two volatilities). With sticky prices, target zones may decrease the conditional standard deviation of the exchange rate relative to free floating while also decreasing the conditional standard deviation of the interest rate differential. Additionally, the volatility of output exhibits positive linkages to the volatility of the exchange rate, which implies that a decrease in the volatility of the exchange rate, brought about by a regime change from free floating to a target zone or by a narrower exchange band, will decrease the volatility of output as well. The lack of such trade-offs can be relevant for the evaluation of alternative exchange rate regimes. Also our inquiry is in line with renewed interest in models with rigidities that have appeared regularly in recent literature, such as Kempa, Nelles, and Pierdzioch (1999) and Beetsma and van der Ploeg (1998).

At an intuitive level, our view of the above research as well as our contribution is along the following lines. "Total volatility" in a closed model—like all the models quoted above including the present—is preserved, pretty much like the preservation of energy in physics. It cannot simply disappear. The negative volatility trade-off recorded by Svensson (1991a) is just a manifestation of the above principle when there is no alternative outlet for the volatility except for the volatilities of the exchange rate and the interest rate differential. When other outlets are available, like the volatility of realignment [Bertola and Svensson (1993)], then the Svensson trade-off might disappear. Something similar happens in the case that we investigate here. Another outlet is available for the volatility, in particular the volatility of the domestic price process. As Kempa, Nelles, and Pierdzioch (1999) documented, target zones weaken the degree of mean reversion of the domestic price level, thus reducing the variability of short-term real interest rates. But weakening of the degree of mean reversion of the domestic price level implies an increase in its volatility.

The paper is organized as follows. Section 2 summarizes the results concerning the volatility of the interest rate differential in the monetary model. Section 3 presents a sticky price exchange rate target zone model, and Section 4 shows that there exists no volatility trade-off under overshooting both with respect to the interest rate differential and the output (as we demonstrate in Section 5). Finally, Section 6 concludes and provides hints towards questions for further research.

2. Volatility Trade-Offs in the Monetary Model

In an economy without price rigidities, where output is always in equilibrium and purchasing power parity holds instantaneously, Svensson (1991) has shown that there is a negative linear trade-off between the conditional standard deviation of the exchange rate and that of the instantaneous interest rate differential in a perfectly credible target zone:

$$\sigma_s(f) + \lambda\sigma_\delta(f) = \sigma\,, \tag{1}$$

where $\sigma_s(f)$ and $\sigma_\delta(f)$ are the standard deviations of the exchange rate and the interest rate differential respectively. They are both functions of f, the fundamental, which will be defined later. σ is the standard deviation of f and λ is the Cagan interest rate semi-elasticity of money demand.

The above follows from the monetary model of exchange rate determination:

$$m - p = \varphi y - \lambda i - v\,, \qquad \varphi,\ \lambda > 0 \tag{2}$$

$$s = p - p^* \tag{3}$$

$$E(ds) = (i - i^*)dt\,, \tag{4}$$

where m is the log of the nominal money supply, p is the log of the price level, y is the log of full employment output, i is the nominal short-term interest rate, s is the log of the exchange rate (domestic over foreign currency), and v is a stochastic disturbance. An asterisk indicates a foreign variable. The above set of equations (2)-(4) represent respectively the money market equilibrium, the purchasing power parity, and the uncovered interest parity condition.

An important assumption is the equation of the stochastic evolution of the disturbance:

$$dv = \mu dt + \sigma dz\,. \tag{5}$$

This is a Brownian motion with drift μ and instantaneous standard deviation σ where dz is a Wiener process. In his seminal paper, Krugman (1991) made the simplest and convenient assumption that v follows a Brownian motion process without drift.

Solving for the exchange rate s one gets

$$s(f) = f + \lambda \frac{E[ds]}{dt}\,, \tag{6}$$

where f is the stochastic fundamental defined as the sum of the money supply and the stochastic disturbance, which is also sometimes referred to as 'exogenous velocity' [Svensson (1992)]:

$$f = m + v\,. \tag{7}$$

An explicit assumption, without loss of generality, has been made that $y = i^* = p^* = 0$ in order to derive the results above. Typically, the money supply is assumed fixed as long as the exchange rate is in the interior of the band and is adjusted when it hits the boundaries.

3. A Model with Price Rigidities

The model is given by the following equations where all the parameters κ, λ, γ, η, φ, and σ are positive real numbers:

$$m - p = \kappa y - \lambda i \tag{8}$$

$$y = -\gamma\left[i - E\frac{(dp)}{dt}\right] + \eta(s - p) \tag{9}$$

$$dp = \varphi(y - \bar{y})dt + \sigma dz \tag{10}$$

$$E(ds) = idt\,. \tag{11}$$

The assumption of price flexibility is replaced by a type of Phillips curve [equation (10)] that relates inflation to the level of excess demand. This is one of the possible ways to model sticky prices [see Sutherland (1994) for alternatives]. We have also assumed that $i^* = 0$ and $p^* = 0$ to simplify expressions.

The only stochastic element is the white-noise disturbance of the inflation process. Demand depends on the real interest rate and the real exchange rate [equation (9)]. Notice that the stochastic differential equation (10) is a general Ito process with variable drift. It is a substantially different model from the one used in Svensson (1991a), as a simple juxtaposition of equations (2) and (5) versus equations (8) and (10) will verify. In fact, it is a form of the Ornstein-Uhlenbeck process with variable mean reversion. The monetary authority intervenes, as in the flex-price model before, infinitesimally and only at the margins in order to defend the band. Thus, the nominal money supply is kept fixed at some target level in the interior of the band and is only adjusted at the boundaries. The cost of modeling the problem this way is that analytical solutions are no longer available, but this is not essential for the purposes of this paper.

4. Interest Rate Differential and Exchange Rate Volatilities

The solution of the target zone model with price inertia for the exchange rate $s(f)$ is

$$s(f) = f(f) + \beta\frac{E[ds(f)]}{dt}, \tag{12}$$

where

$$f = \zeta p + \vartheta \tag{13}$$

$$\beta = \frac{\lambda + \gamma(\kappa - \varphi\lambda)}{\eta\kappa} \tag{14}$$

$$\zeta = \frac{\varphi\gamma + \eta\kappa - 1}{\eta\kappa} \tag{15}$$

$$\vartheta = \frac{1 - \varphi\gamma}{\eta\kappa}m + \frac{\gamma\varphi}{\eta}\bar{y}\,. \tag{16}$$

A unique converging equilibrium saddle path is guaranteed for $\beta > 0$. This basically represents the condition under which the deterministic system (obtained by setting $\sigma = 0$) possesses a stable saddlepoint equilibrium [Miller and Weller (1991, p.1382)]. Thus for the remainder of the paper we make this assumption.

One can also distinguish two cases with respect to the exchange rate response to money supply changes. If $\kappa\eta + \varphi\gamma < 1$ [obtained by setting $ds/dm > 1$, Miller and Weller (1991)], the exchange rate overshoots. If the inequality holds in the opposite direction, the exchange rate undershoots. The analysis that follows assumes overshooting while the undershooting case retains the well-known trade-offs of the flex-price model.

The exchange rate turns out to be a decreasing function of the fundamental whose slope is less than one in absolute value [Miller and Weller (1989)].

We now define the instantaneous interest rate differential. Since $i^* = 0$, it is given by

$$\delta = i \, . \tag{17}$$

Using the interest rate parity equation (11) and equation (12) we get

$$\delta(f) = \frac{s(f) - f}{\beta} \, . \tag{18}$$

Differentiating with respect to f we obtain

$$\delta'(f) = \frac{s'(f) - 1}{\beta} \, . \tag{19}$$

The negativity of the derivative of the interest rate differential above is guaranteed since the slope of the exchange rate is negative.

Now, since both the interest rate differential and the exchange rate are functions of the fundamental f, generalized Ito processes can describe their stochastic evolution as follows:

$$ds(f) = \mu_s(f)dt - \sigma_s(f)dz \tag{20}$$

$$d\delta(f) = \mu_\delta(f)dt - \sigma_\delta(f)dz \, . \tag{21}$$

Writing the equations above this way guarantees positive standard deviations since both functions are decreasing in the fundamental.

From equation (13) we derive the stochastic evolution of the fundamental

$$df = \zeta dp = \zeta\theta(y - \bar{y})dt + \zeta\sigma dz \, . \tag{22}$$

Now, using Ito's rule, the diffusion coefficients of the two differentials are

$$\sigma_\delta(f) = -\delta'(f)\zeta\sigma \tag{23}$$

$$\sigma_s(f) = -s'(f)\zeta\sigma \, . \tag{24}$$

Substituting equations (23) and (24) into (19) we have

$$\beta\sigma_{\delta}(f) = \sigma_{s}(f) + \sigma\zeta \ . \tag{25}$$

And obviously,

$$\frac{d\sigma_{\delta}(f)}{d\sigma_{s}(f)} = \frac{1}{\beta} > 0 \ . \tag{26}$$

We have shown that no trade-off exists between the volatility of the exchange rate and that of the nominal interest rate differential in the particular case we studied above. Thus a policy that narrows the exchange rate fluctuation margins decreases the conditional volatility of both the exchange rate and the interest rate differential. This implication is in fact consistent with the empirical findings of Flood, Rose, and Mathieson (1991) since they do not find any negative trade-offs for the EMS countries.

5. Output and Exchange Rate Volatilities

We now turn our attention to the conditional standard deviation of output, and we show that it is positively related to that of the exchange rate as well. Solving the model for the output y we get

$$y(f) = -\left[\frac{1}{\kappa\zeta} + \frac{1}{\kappa\beta}\right] f + \frac{\lambda}{\beta\kappa} s(f) + A \ , \tag{27}$$

where

$$A = \left[\frac{1}{\kappa} - \frac{1-\varphi\gamma}{\kappa^{2}\zeta\eta}\right] m + \frac{\gamma\varphi}{\eta\kappa\zeta}\bar{y} \ . \tag{28}$$

Differentiating equation (27) yields

$$y'(f) = \xi s'(f) - \xi - \frac{1}{\kappa\zeta}, \ \xi = \frac{\lambda}{\kappa\beta} > 0 \ . \tag{29}$$

The above expression is always negative since the exchange rate is decreasing in the fundamental. Since y is a function of f, the following Ito process will give its dynamics:

$$dy(f) = \mu_{y}(f)dt - \sigma_{y}(f)dz \ . \tag{30}$$

Now from Ito's rule

$$\sigma_y(f) = -y'(f)\zeta\sigma \tag{31}$$

$$\sigma_s(f) = -s'(f)\zeta\sigma \ . \tag{32}$$

Substituting (31) and (32) into (29) we get

$$\sigma_y(f) = \xi\sigma_s(f) + v\sigma\zeta, \ v = \xi + \frac{1}{\kappa\zeta} > 0 \ . \tag{33}$$

And finally,

$$\frac{d\sigma_y}{d\sigma_s} = \xi > 0 \ . \tag{34}$$

With sticky prices, and when the exchange rate overshoots, a decrease in the conditional volatility of the exchange rate leads to a decrease in the conditional volatility of output. The results of Weber (1990) seem to indicate that the unconditional volatility of output is lower for EMS countries. It would be interesting to obtain measures of the conditional output volatility in future research.

One can also show that the results of the first generation target zone models, based on the monetary model, can be viewed as a special case of the sticky price model and can in fact be obtained as the speed of adjustment in the economy, φ, increases. As φ rises, prices adjust faster to shocks and the conditions for undershooting are eventually met, in which case volatility trade-offs are the same as the ones we observe in the monetary model.

6. Conclusions

We have shown that there exists a positive linear linkage between the volatility of the exchange rate and the volatility of the instantaneous interest rate differential in an economy with sticky prices. The linear negative trade-off relationship derived by Svensson (1991) for the flex-price model, depends on the assumption of fast adjusting prices. When prices are sticky and the exchange rate overshoots, no such trade-off exists.

The implications of the sticky price model might be more consistent with EMS data than the monetary model results. The empirical evidence in Flood, Rose and Mathieson (1991) seems to indicate that non-linearities in the exchange rate-fundamental relationship do not exist. Underlying their entire analysis, though, is the monetary model and an estimate of the interest semi-elasticity of money demand of 0.1, which determines the degree of non-linearity in the exchange rate. Notice that for $\lambda = 0$, $s = f$, and the exchange rate-fundamental relationship is linear. As λ increases, the relationship becomes non-linear. In our model it is β that determines the degree of non-linearity, and this parameter can reasonably be greater than 0.1.

Concerning volatility trade-offs, their tests seem to indicate that, for the EMS, no negative volatility trade-offs exist between exchange rates and the interest rate differential. These findings are consistent with the sticky price model that we presented here even in the absence of realignment risk.

Finally, taking into account our results allows one to view the work of Kempa, Nelles, and Pierdzioch (1999) from a slightly different angle. They use a model with price rigidities to study the effect of imposing exchange rate target zones on the term structure of interest rates. Using qualitative arguments and simulations, they show that target zones weaken the degree of mean reversion of the domestic price level, thus reducing the variability of short-term real interest rates. Thus volatility may be alternatively transferred to prices, since they become less mean reverting, while both the volatility of the exchange rate and the volatility of the interest rate differential decrease.

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