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Investment Decisions and Normalization with Incomplete Markets: A Pitfall in Aggregating Shareholders' Preferences

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Abstract

Profit maximization is not a well defined objective when markets are incomplete. Several criteria of investment choice have therefore been put forward in the literature, some of which crucially hinge upon aggregation of shareholders' preferences, as is the case with the criteria proposed by Drèze (1974) and Grossman and Hart (1979). This note shows that these criteria are normalization dependent, i.e., their outcome depends on the good chosen to express individuals' marginal rates of substitution.

Key words: investment decisions; normalization; incomplete markets

JEL classification: D52; D70; D81

1. Introduction

The usual assumption about firms in a competitive, frictionless economy is that they act in the interest of shareholders. The meaning of this assumption is clear when all commodities (at all possible dates and/or states of nature) are priced and exchanged against each other, as in the model of general equilibrium of Arrow and Debreu (1954) and McKenzie (1954), where all agents maximize their preferences subject to a unique, intertemporal budget constraint and firms simply have to maximize profits in order to maximize shareholders' wealth. As all goods (in an extended sense) are priced, the concept of profit is well defined, and unanimity of shareholders is guaranteed. When markets are incomplete, on the other hand, agents can only use their personal, subjective prices (i.e., their marginal rates of substitution at different date-event pairs) to perform this evaluation. This normally implies that the evaluations of investment plans will be different across shareholders, which creates an important problem for defining a sensible objective function for firms.

One possible approach to the problem consists in deriving firms' behaviour

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from shareholders' preferences, as reflected in individual state prices, in the following way: firms using a convex combination of shareholders' marginal utilities of income (of initial shareholders, as in Grossman and Hart, 1979, or final shareholders, as in Drèze, 1974) to evaluate profits. A generalization of this approach, defined as "value maximizing," is proposed in De Marzo (1993).

In this note we focus on this well known approach and show how the criteria that have been put forward to solve the indeterminacy problem are themselves indeterminate, in the sense that they depend on the particular normalization which is adopted. This is reminiscent of a similar problem encountered in the literature on imperfect competition (see Gabszewicz and Vial, 1972) but with a remarkable difference: the problem arises, in the case of incomplete markets, only when one tries to aggregate shareholders' preferences, one way or another. In the case of imperfect competition, in contrast, the problem arises even in the case of a single ownership.

The rest of the work is organized as follows. Section 2 contains some brief remarks on the problem of asset valuation with and without complete markets. Section 3 inserts this problem in the context of a heterogeneous agent problem and illustrates the so-called "shareholder constrained efficient" rule for selecting investment decisions on the part of firms. Section 4 shows that such a rule, as any other depending on the aggregation of agents' preferences, crucially depends on the normalization of preferences. Section 5 concludes with an eye to applications.

2. Asset Valuation with Incomplete Markets

Let us consider the following simple exchange economy, extending over T+1 periods and featuring only one good per period. There is also one asset, which pays fixed quantities (a vector y) of the good at the various dates. The asset can be traded at one date only (i.e., it is not re-traded).

Suppose an agent h in this economy solves the problem:

$$\max_{x^h,\theta^h} \ u^h(x^h) \tag{2.1}$$

$$\text{subject to} \quad p_0 x_0^h + (q - p_0 y_0)\theta^h = p_0 w_0^h + q\bar{\theta}^h$$

$$p_t x_t^h = p_t w_t^h + p_t y_t \theta^h, t = 1, ..., T \ ,$$

where $x = (x_0, x_1, \ldots, x_t, \ldots, x_T)$ is the vector of consumptions at various dates, $y = (y_0, y_1, \ldots, y_T)$ is the vector of returns of an investment (asset) from period 0 to period T, $\bar{\theta}^h$ and θ^h denote respectively the initial and final holdings of the asset, q is the asset price, and p_0 and p_t are the spot prices for the physical good available at date 0 and at dates $t = 1, \ldots, T$.

Spot prices at all dates other than the first play no role as there is only one good per period and trade does not occur; therefore they are all normalized to one, as is the spot price of the good at time 0. Notice, moreover, that the purchase of the asset takes place at time 0, which is tantamount to assuming that the opportunity cost of buying the asset is expressed in terms of forgone consumption at date 0. The first order condition of this problem with respect to the asset is:

$$\frac{\partial u^h}{\partial x_0^h} q = \frac{\partial u^h}{\partial x_0^h} y_0 + \frac{\partial u^h}{\partial x_1^h} y_1 + \frac{\partial u^h}{\partial x_2^h} y_2 + \ldots + \frac{\partial u^h}{\partial x_T^h} y_T \tag{2.2}$$

or

$$q = y_0 + (\frac{\partial u^h}{\partial x_1^h} / \frac{\partial u^h}{\partial x_0^h}) y_1 + (\frac{\partial u^h}{\partial x_2^h} / \frac{\partial u^h}{\partial x_0^h}) y_2 + \ldots + (\frac{\partial u^h}{\partial x_t^h} / \frac{\partial u^h}{\partial x_0^h}) y_t + \ldots + (\frac{\partial u^h}{\partial x_T^h} / \frac{\partial u^h}{\partial x_0^h}) y_T. \tag{2.3}$$

At an interior optimum the price of the asset has to be equal to the weighted average of its returns at the various dates, the weights being the components of the gradient of the utility function, divided by the marginal utility of consumption at time 0. Such weights might be considered as “subjective discount rates” applicable to income streams received at various dates. The right hand side of the previous expression can globally be taken to represent the marginal willingness to pay for the asset, which is expressed in terms of units of consumption at date 0 (which should also be clear by the fact that y_0 is not discounted by any factor). There is no compelling reason, however, for expressing agent *h*’s marginal willingness to pay in terms of consumption at date 0. We might wish to express this in terms of consumption at any time *t*. In this case the first order condition for a maximum would read:

$$q = (\frac{\partial u^h}{\partial x_0^h} / \frac{\partial u^h}{\partial x_t^h}) y_0 + (\frac{\partial u^h}{\partial x_1^h} / \frac{\partial u^h}{\partial x_t^h}) y_1 + \ldots + y_t + \ldots + (\frac{\partial u^h}{\partial x_T^h} / \frac{\partial u^h}{\partial x_t^h}) y_T , \tag{2.4}$$

where the discount factor attached to date *t* is obviously equal to one.

In fact, expression (2.4) can be easily recovered from solving the problem:

$$\max_{x^h, \theta^h} \ u^h(x^h) \tag{2.5}$$

$$\text{subject to} \quad x_t^h + (q - y_t)\theta^h = w_t^h + q\overline{\theta}^h$$

$$x_s^h = w_s^h + y_s \theta^h, s \neq t ,$$

where all spot prices have been normalized to one.

The difference between problem (2.1) and (2.5) should be quite clear: in the former the price of the asset is paid in period 0, while in the latter it is paid in period *t*.

The marginal willingness to pay for an asset, i.e., the right hand sides of expressions (2.3) and (2.4), can also be interpreted in terms of an agent’s valuation of a small additional quantity of this particular asset in his portfolio. This is why it has been often used to evaluate marginal changes of asset returns with respect to a given vector of returns.

3. Investment Decisions with Heterogeneous Agents

Consider now a world with many, heterogeneous agents who differ in terms of utilities and/or endowments. Each of the agents assesses the value of an asset as illustrated in the previous section, i.e., by computing the weighted average of the asset's returns, the weights being his/her marginal rates of substitution.

Let us now consider the problem of the manager of a firm who has to decide on the best investment plan to implement. His problem might be stated in the following way:

$$\max_{y} \pi' y$$
$$\text{subject to} \quad y \in Y ,$$

where $y = (y_0, \ldots, y_T)$ is a production vector, π is a suitable price vector (therefore $\pi' y$ corresponds to the profit associated with the investment plan y), and Y is the firm's production possibility set.

The manager is therefore confronted with the problem of finding a suitable vector of prices to evaluate revenues accruing in different time periods, i.e., a suitable vector of discount factors. These could be provided by the market itself were an adequate set of financial instruments available; this would indeed be the case, for instance, if a sufficient number (one for every time period, minus one) of futures were available.

When such a complete set of real or financial instruments is missing, however, markets do not provide sufficient indications for unambiguously evaluating revenues accruing to the firm in the various time periods. This does not create any problem if the firm is a sole ownership, as it can easily be shown that choosing a production plan which maximizes the owner's utility function is tantamount to maximizing profits using as prices the owner's marginal rates of substitution, which excellently serve the purpose of implicit prices.

The picture gets confused, however, if the firm is owned by many agents because, when markets are incomplete, there is no reason why marginal rates of substitution, and therefore marginal valuations of assets, should ever coincide across agents. In other words, normalized gradients (or "subjective discount rates") will not generically coincide across agents, as was shown in the contributions by Duffie and Shafer (1985) and DeMarzo (1993).

What can managers in a corporation do, in such a case? Whose marginal rates of substitution should they use to evaluate alternative investment plans? Many possible answers have been put forward in the literature as to the possible criteria that a corporation might implement. One such criterion is that of "shareholder constrained efficiency," introduced by Drèze (1974), modified by Grossman and Hart (1979), and generalized by Geanakoplos et al. (1990) and De Marzo (1993), which basically consists in evaluating alternative investment plans using a weighted average of shareholders' discount rates, the weights being given by shareholders' ownership quotas.

This investment criterion has been shown to yield investment decisions y^* which are "shareholder constrained efficient," in the sense that no group (possibly singletons) of shareholders would propose a marginal change of the plan y^* if they had to compensate all the shareholders suffering a loss from the change. It seems evident that a notion of veto power is embodied in the definition (but one could obviously think of a "*k*-percent" shareholders' constrained efficiency definition).

In the following section it will be shown, by means of a simple example, that the outcome of such a rule is normalization dependent, in the sense that it yields different results (i.e., different equilibrium allocations and prices) when different normalization rules are used (i.e., marginal rates of substitutions are computed in terms of different benchmark consumption). This is the case, of course, only when markets are incomplete.

4. Normalization Matters

Let us consider the following simple example consisting of one firm owned by two shareholders living for three periods. The intertemporal technology of the firm is linear of the following type:

$$Y = \left\{ y \in \Re^3; y_0 = -k, y_1 = -\frac{1}{2}k, y_2 = 2\sqrt{k}, k \geq 0 \right\}.$$

Therefore, this firm produces a positive output in period 2 using inputs in the other two periods. There is only one good per period, and we will refer equivalently to good 0, 1, and 2 to indicate, respectively, consumption in period 0, 1, and 2. Markets are incomplete in that there are no assets which allow agents to redistribute freely their wealth across time periods.

Shareholders 1 and 2 own quotas of respectively θ and $1-\theta$ of this firm. They have preferences represented by the utility functions

$$u^1 = x_0^1 + 2x_1^1 + x_2^1$$
$$u^2 = x_0^2 + x_1^2 + 3x_2^2 ,$$

and they have no initial endowments.

The linearity of this example has the only purpose of simplifying calculations and entails no loss of generality. We only intend to show that "utility maximizing" rules, like those illustrated above, generate outcomes which depend on the particular normalization which has been adopted, and this can be more easily demonstrated in our simplified setup where shares are not traded.

To find out the "shareholder efficient production plan" in the case of normalization with respect to good 0, it is enough to solve the following optimization plan:

$$\max_k \pi' y ,$$

where the price vector π is given by a weighted average of shareholders' marginal rates of substitution, i.e.,

$$\pi = \theta \times MRS^1 + (1-\theta) \times MRS^2 ,$$

evaluated at the equilibrium allocation.

It is easy to realize that, given linear utilities as in our case, marginal rates of substitution do not depend on the particular equilibrium allocation. It is therefore straightforward to solve the firm's maximization problem.

Firstly, let us consider agents' marginal rates of substitutions expressed in terms of consumption at date 0, which yields the following expression of profits:

$$\pi' y = -k - (1/2)(1+\theta)k + 2(3-2\theta)\sqrt{k}. \tag{4.0}$$

By computing the first derivative of (4.0) with respect to *k*, equating it with 0, and solving the resulting equation in *k*, we obtain the following optimal plan, as a function of θ:

$$k = \left[(3-2\theta)/(1+(1+\theta)/2)\right]^2 . \tag{4.1}$$

This is, in fact, the "efficient" plan, were compensation (mentioned in the previous paragraph) to take place in terms of consumption at date 0.

If, alternatively, the utility gradients were normalized with respect to consumption in period 1, the optimal decision would be:

$$k = \left[(6-5\theta)/(3-\theta)\right]^2 . \tag{4.2}$$

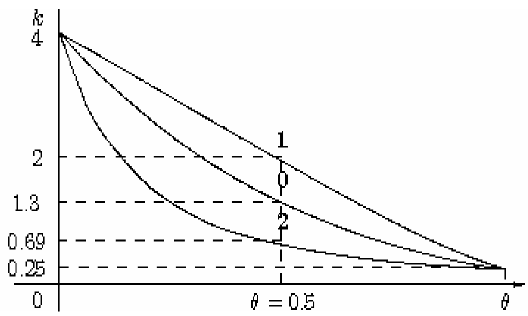
Finally, if gradients were normalized with respect to consumption at time 2, we would obtain:

$$k = \left[2/(1+3\theta)\right]^2 . \tag{4.3}$$

It is evident that the three normalization rules lead to three different outcomes. Figure 1 shows that the three rules can even be uniformly ranked according to the optimal level of investment, *k*, as a function of the ownership quota, θ (the boldface number above each curve indicates the normalization adopted in the computation).

What drives the result is that agents differ in marginal rates of substitution between consumption at different dates. Let us consider, for example, the shareholders' efficient plan corresponding to θ = 0.5, when gradients are normalized with respect to the good available in period 0. From (4.1) we obtain k = 64/49. If the gradients of the utility functions were normalized with respect to consumption in period 1, the sum of the marginal willingness to pay would be positive and equal to 0.281. The reason for this is very clear. For the second agent, nothing changes with respect to the first case: his/her marginal utility for period 1 consumption is, in fact, equal to that for consumption at time 0. This implies, in turn, that a marginal increase in k is equivalent, from agent 2's standpoint, to the same additional quantity of good 0 and good 1. For agent 1, in contrast, the marginal utility of consumption at date 1 is twice as much as that of consumption at date 0, which means that he/she will demand, in terms of good 1, half of the compensation demanded in terms of good 0. That's why agent 2 is left with a positive surplus, which prevents $k = 64/49$ from being a shareholders' constrained efficient plan. In fact, and this should be clear from Figure 1, the shareholder constrained efficient plan corresponding to $\theta = 0.5$ is associated with a higher k.

Figure 1.



Quite the contrary occurs when the gradients of utility functions are normalized with respect to consumption in period 2; in this case it becomes much more expensive for the second agent to compensate the first for his marginal loss; therefore the sum of agents' valuations of a marginal change in production becomes negative and equal to −0.375. As a consequence of this, we can observe from Figure 1 that the shareholder efficient plan is associated with a lower value of k. Needless to say, if marginal rates of substitution were equal across agents, as in the case of complete markets, the three possible normalizations would lead to exactly the same results.

It is also very easy to show that for each agent the three normalization rules can be uniformly ranked in terms of utility levels.

5. Final Remarks

One of the most interesting and controversial issues in the literature about incomplete markets concerns the behaviour (in terms of investments) of competitive corporations. If markets are incomplete, the usual goal of profit maximization becomes ambiguous in that marginal rates of substitution across date-event pairs are not equalized across agents. Many criteria have been proposed that are based on the aggregation of individuals' marginal rates of substitutions, such as the so-called Drèze (1974) criterion, the Grossman-Hart (1979) criterion, or the generalizations proposed by DeMarzo (1993). In this note it is explained, and shown by means of a simple example, that all those criteria are normalization dependent in that the investment plan selected by firms depends on the particular normalization chosen for individuals' marginal rates of substitution. This result, which might in itself be taken as little more than a theoretical curiosum, bears important implications for investment selection procedures. Suppose, for instance, that discounted cash flow (DCF) techniques were used by a corporation to select investment plans. As shareholders' discount rates generically differ when markets are incomplete, managers might try to reach a consensus by resorting to an "average" (in the sense made precise above) discount rate to evaluate future profit streams. In this case the selected investment plan would crucially depend on the particular vantage point taken in the discounting procedure. However, the optimal selection would be totally independent of the vantage point if markets were complete. Any accounting rule which played a role in determining the structure of a DCF analysis would therefore also impact the optimal investment selection procedure in a realistic context of market incompleteness. In this respect, to go back to a well known question raised by Plott and Sunder (1981), we might conclude that when markets are incomplete there are remarkable reasons why accounting should matter.

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