A Note on Dual Hedging

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Abstract

Under current Internal Revenue Services guidelines, gains from futures contracts serving price (quantity) risk management purposes are treated as ordinary (capital) income. This paper finds that, although dual hedging opportunities are available, the asymmetric tax treatment prevents firms from trading "quantity" futures contracts.

Key words: dual hedging; ordinary income; capital income

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1. Introduction

For most commodities, futures contracts are available for hedgers to reduce price risk. There are also contracts based upon a production index that can be applied to reduce quantity risk faced by a hedger. In this note, we call the former contract a price contract and the latter a quantity contract. For example, the corn futures contract traded at the Chicago Board of Trade (CBOT) is a price contract and the Iowa corn yield futures contract traded at the same exchange is a quantity contract. When both contracts are available, a producer may trade on the two futures markets to reduce his profit risk. This operation is called "dual hedging."

Vukina et al. (1996) adopted a mean-variance approach to determine the optimal hedging strategy using a quantity futures contract. Li and Vukina (1998) considered how a corn producer may reduce his profit risk by trading in both corn price and corn yield futures contracts. The analysis is, once again, restricted to the mean-variance framework.

This note extends the work of Li and Vukina (1998) in two directions. First, we consider a general expected utility analysis incorporating the mean-variance analysis as a special case. Secondly, we introduce the tax rules into the model. Specifically, the current guidelines issued by Internal Revenue Services offer asymmetric treatment between gains (and losses) from a price futures contract and gains (and losses) from a quantity futures contract. Gains or losses from a price futures position (estab-

lished to reduce the price risk) are classified as ordinary and can be applied to offset gains (or losses) from the spot position. On the other hand, gains or losses from a quantity futures position are classified as capital and cannot offset losses (or gains) from the spot position.

The main contribution of this paper is to demonstrate an irrelevance result that a hedger will choose not to trade the quantity contract. This result is consistent with empirical observations. For example, the Iowa corn yield futures contract was launched at the CBOT in 1995. The numbers of contracts traded declined from 8,648 in 1995 to 1,986 in 1996 and 832 in 1997. Three additional corn yield futures contracts were introduced in 1997: U.S., Ohio, and Nebraska. For the first year, the trading volumes were 870, 142, and 78, respectively. In 1997, the numbers increased to 182 and 1,730 for Ohio and U.S. corn yield futures, respectively. The number for Nebraska corn yield futures, however, fell to 40. By way of comparison, the trading volumes for the CBOT corn price futures from 1995 to 1997 were 33.9, 44.5, and 39.1 millions, respectively (*http://www.botcc.com/about/clearance.html*).

2. The Basic Model

We consider a one-period framework. At the beginning of the period, say, time $t = 0$, the firm commits to a constant-return-to-scale technology such that production cost per unit is c. The price and output are realized at the end of the period, i.e., time $t = 1$, and denoted by P and Q, respectively. The before-tax profit from production is

$$\pi = (P - c)Q . \tag{1}$$

At $t = 0$, both P and Q are random variables. The firm therefore incurs risk in its profit. To reduce the risk, the firm may trade both price and quantity futures contracts at $t = 0$. Assume that the firm sells x price futures contracts at time 0 and liquidates the position at time $t = 1$. Let f_t denote the price of the price futures contract at time t, $t = 0$, 1. The profit from trading in the price futures contract is $(f_0 - f_1)x$. Similarly, assume that the firm sells y quantity futures contracts at time 0 and liquidates the position at time 1.

The quantity futures contract is priced on a quantity index. As an example, in corn yield futures it would be the bushels of corn produced in the contract-specified area. Let $\bar{Q}$ denote the index of the quantity futures contract at time 0. In trading, the index must be converted to some monetary unit. Suppose that each unit of output is converted to one dollar. For example, one bushel is converted to one dollar. Note that the conversion is for trading purposes; it does not imply one bushel of corn is worth one dollar. The firm's profit from trading in the quantity futures contract is $(\bar{Q} - Q)y$.

The current tax codes classify income from trading in the price futures contract as ordinary; income from trading in the quantity futures contract as a capital gain or loss. For simplicity, we assume both incomes are subject to the same tax rate, τ. Consequently, the firm's after-tax profit is as follows:

$$\pi^A = (P-c)Q + (f_0 - f_1)x - \tau\ Max\{0,(P-c)Q+(f_0-f_1)x\} \qquad (2)$$
$$+(\bar{Q}-Q)y - \tau\ Max\{0,(\bar{Q}-Q)y\}.$$

Note that production profit can be applied to offset the losses from trading in the price futures contract, but not those from trading in the quantity futures contract.

We assume the firm has a strictly concave utility function, $U(\pi^A)$. At time 0 the firm chooses the optimal positions in the price and quantity futures markets, x^* and y^*, to maximize the expected utility at time 1, $EU(\pi^A)$, where the expectation operation is taken with respect to random variables, P, Q, and f_1. To proceed, we assume that both futures markets are unbiased, implying $E(f_1) = f_0$ and $E(Q) = \bar{Q}$. Also, the price futures contract is assumed to be a perfect hedge instrument such that $P = f_1$ so that the spot position at the maturity date can offset the futures position at no cost.

Note that, in our model, the firm only makes financial decisions (i.e., takes the two futures positions), while production is exogenously given. This approach is well established in the futures market literature; see, for examples, Vukina et al. (1996) and Li and Vukina (1998). A possible justification is as follows. Suppose that production takes a long period of time to complete. At the time when the amount of input is chosen, the futures markets (that mature at the production completion date) are either unavailable or very thinly traded. Therefore, the firm will adopt a sequential approach, to choose the input (and, correspondingly, the expected output) first and the futures positions in subsequent periods.

3. The Main Result

Suppose that $c = f_0$, i.e., the futures price at time 0 is the same as the unit cost. Note that in our model we consider only a producer who will short futures contracts. We do not model the behavior of long hedgers and speculators. As long as there exist sufficiently many risk neutral speculators, the futures price at time 0 will equal the unit cost. In addition, we have $P - c = f_1 - f_0$ which implies $E(P-c) = 0$. That is, the firm is expected to earn normal profit only. Alternatively, one can reinterpret c as the marginal production cost plus the risk premium. The assumption then states that the firm cannot earn excess profit beyond the risk premium.

Let $\varepsilon = f_1 - f_0$, the change in the price of price futures contract, and let $\theta = Q - \bar{Q}$. Moreover, let $z = x - \bar{Q}$, the deviation of the futures position in the quantity contract from the expected output level. Then the after-tax profit can be rewritten as follows:

$$\pi^A = \varepsilon\theta - \varepsilon z - \tau\ Max\{0,\varepsilon\theta - \varepsilon z\} - \theta y - \tau\ Max\{0,-\theta y\}.$$

Let $g(\varepsilon,\theta)$ denote the joint probability density function of ε and θ. We consider the case of a symmetric joint density. This assumption implies that price and quantity are uncorrelated. In a competitive market, each producer has no impact

on the market price. Therefore, the (market) price and the production quantity (of each individual producer) are uncorrelated. Given this assumption, one may argue that the full hedging result (in price futures market) is immediate from McKinnon (1967). This conjecture is, however, invalid. The reason is that our model incorporates nonlinear taxation. Correlation, however, serves to describe a linear relationship between two random variables. Consequently, non-correlation is not sufficient to derive the full hedging result. The following theorem provides our main result. The proof is shown in the Appendix.

Theorem 1: Assume that $g(\varepsilon,\theta)$ is symmetric such that $g(\varepsilon,\theta) = g(-\varepsilon,-\theta)$. Then the optimal futures positions are $x^* = \bar{Q}$ and $y^* = 0$. That is, the firm should hedge the amount of expected output in the price futures contract and not engage in quantity futures contract trading at all.

4. Conclusion

The main theorem provides a no hedging result for the quantity futures contract. This result is similar to Pirrong (1995), Lien (1999), and Milevsky and Prisman (1999) where only the price futures contract is available. Specifically, when the gains or losses from futures trading are classified as capital gains or losses and cannot be applied to offset gains or losses from business operations, quantity futures contracts do not provide any hedging function. As a result, the firm will elect not to trade quantity futures contracts at all. On the other hand, gains or losses from trading in the price futures contract are classified as ordinary and therefore eligible to offset gains or losses from business operations. When the futures market is unbiased, we get the well-known full hedge result; namely, the optimal futures position equals the expected output level. Our theorem demonstrates that the results can be generalized to the case in which both price and quantity contracts are available. Due to asymmetric taxation, the firm's optimal hedging policy is a full hedge on the price futures contract and no trading on the quantity futures contract.

Appendix

Proof of Theorem 1:

We consider first the case that $y \geq 0$ and $z \geq 0$. Depending on the values of ε and θ, there are six possible regimes:

Regime 1. $\varepsilon \geq 0$ and $\theta \geq z \geq 0$. Herein, $\pi^A = (\varepsilon\theta - \varepsilon z)(1-\tau) - \theta y$.
Regime 2. $\varepsilon \geq 0$ and $z \geq \theta \geq 0$. Herein, $\pi^A = (\varepsilon\theta - \varepsilon z) - \theta y$.
Regime 3. $\varepsilon \geq 0$ and $z \geq 0 \geq \theta$. Herein, $\pi^A = (\varepsilon\theta - \varepsilon z) - (1-\tau)\theta y$.
Regime 4. $\varepsilon \leq 0$ and $\theta \geq z \geq 0$. Herein, $\pi^A = (\varepsilon\theta - \varepsilon z) - \theta y$.
Regime 5. $\varepsilon \leq 0$ and $z \geq \theta \geq 0$. Herein, $\pi^A = (\varepsilon\theta - \varepsilon z)(1-\tau) - \theta y$.
Regime 6. $\varepsilon \leq 0$ and $z \geq 0 \geq \theta$. Herein, $\pi^A = (\varepsilon\theta - \varepsilon z)(1-\tau) - (1-\tau)\theta y$.

As a consequence, the expected utility is the sum of six components, each corresponding to a different regime. For example, the component corresponding to regime 1 is

$$\int_0^\infty \int_z^\infty U[\varepsilon(\theta-z)(1-\tau)-\theta y]g(\varepsilon,\theta)d\theta d\varepsilon \, ,$$

and the component corresponding to regime 6 is

$$\int_{-\infty}^0 \int_{-\infty}^0 U[\varepsilon(\theta-z)(1-\tau)-(1-\tau)\theta y]g(\varepsilon,\theta)d\theta d\varepsilon \, .$$

Upon taking the partial derivative with respect to z and then evaluating the result at $y = z = 0$, we arrive at the following expression:

$$\left. \frac{\partial EU(\pi^A)}{\partial z} \right|_{y=z=0} = A_1 + A_2 + A_3 + A_4 \, , \tag{3}$$

where

$$A_1 = \int_0^\infty \int_0^\infty -U'[\varepsilon\theta(1-\tau)]\,\varepsilon(1-\tau)g(\varepsilon,\theta)d\theta d\varepsilon \tag{4}$$

$$A_2 = \int_0^\infty \int_{-\infty}^0 -U'(\varepsilon\theta)\varepsilon g(\varepsilon,\theta)d\theta d\varepsilon \tag{5}$$

$$A_3 = \int_{-\infty}^0 \int_0^\infty -U'(\varepsilon\theta)\varepsilon g(\varepsilon,\theta)d\theta d\varepsilon \tag{6}$$

$$A_4 = \int_{-\infty}^0 \int_{-\infty}^0 -U'[\varepsilon\theta(1-\tau)]\,\varepsilon(1-\tau)g(\varepsilon,\theta)d\theta d\varepsilon \, . \tag{7}$$

Note that $U'(.)$ denotes the first derivative with respect to π^A. By symmetry of the joint density function, $A_4 = -A_1$ and $A_3 = -A_2$. Therefore, equation (3) becomes:

$$\left. \frac{\partial EU(\pi^A)}{\partial z} \right|_{y=z=0} = 0 \, . \tag{3'}$$

Similarly, if we take the partial derivative of $EU(\pi^A)$ with respect to y and then evaluate the result at $y = z = 0$, we derive the following:

$$\left. \frac{\partial EU(\pi^A)}{\partial y} \right|_{y=z=0} = B_1 + B_2 + B_3 + B_4 \, , \tag{8}$$

where

$$B_1 = \int_0^\infty \int_0^\infty -U'[\varepsilon\theta(1-\tau)]\,\theta g(\varepsilon,\theta)d\theta d\varepsilon \tag{9}$$

$$B_2 = \int_0^\infty \int_{-\infty}^0 -U'(\varepsilon\theta)\theta(1-\tau)g(\varepsilon,\theta)d\theta d\varepsilon \tag{10}$$

$$B_3 = \int_{-\infty}^0 \int_0^\infty -U'(\varepsilon\theta)\theta g(\varepsilon,\theta)d\theta d\varepsilon \tag{11}$$

$$B_4 = \int_{-\infty}^0 \int_{-\infty}^0 -U'[\varepsilon\theta(1-\tau)]\theta(1-\tau)g(\varepsilon,\theta)d\theta d\varepsilon . \tag{12}$$

By symmetry of the joint density function, $B_4 = -(1-\tau)B_1$ and $B_2 = -(1-\tau)B_3$. Therefore, equation (8) becomes

$$\left.\frac{\partial EU(\pi^A)}{\partial y}\right|_{y=z=0} = \tau(B_1 + B_3) \leq 0. \tag{8'}$$

In other words, within the range $y \geq 0$ and $z \geq 0$, $y = z = 0$ is the optimal solution.

Similar analyses can be applied to the other three cases: $y \geq 0$ and $z \leq 0$, $y \leq 0$ and $z \geq 0$, and $y \leq 0$ and $z \leq 0$. In each case, $y = z = 0$ is the optimal solution. Consequently, we have $y^* = z^* = 0$, implying $x^* = \overline{Q}$.

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