

An Intertemporal Pasinettian Model with Government Sector

João Ricardo Faria*

School of Social Sciences, University of Texas at Dallas—Richardson, U.S.A.

Ricardo Azevedo Araujo

Catholic University of Brasilia, Brazil

Abstract

This paper analyses the relevance of the Cambridge equation in the presence of government when the assumption of fixed savings is relaxed. We consider an intertemporal representative agent model with Pasinettian features. The results are: (i) the equilibrium distribution of income between wages and profits, as stated by the Cambridge equation, is not affected by the occurrence of sustained deficits or surpluses, (ii) the rate of profit is not determined by the Cambridge equation, and (iii) only taxation on profits affects the profit rate and, as a consequence, capital accumulation, wages, and output.

Key words: intertemporal choice; factor income distribution

JEL classification: D91; D33

1. Introduction

One of the controversies involving the Cambridge equation (also known as Pasinetti paradox after Samuelson and Modigliani, 1966) is related to the role of a government sector in Kaldor-Pasinetti models of growth and income distribution. By introducing government spending and taxation in this framework, Steedman (1972) obtained two important results under the assumption of a balanced government budget: (i) the “Dual Theorem” was proved to be irrelevant and (ii) the Cambridge equation remained valid and was formulated as:

$$r = \frac{n}{s_c(1-t_p)}, \quad (1)$$

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*Correspondence to: School of Social Sciences, University of Texas at Dallas, P.O. Box 830688, GR 31, Richardson, TX 75083-0688, U.S.A. E-mail: jocka@utdallas.edu. The authors thank an anonymous referee for useful suggestions.

where r is the rate of profit, n is the natural growth rate, which is assumed to be equal to the growth rate of the population, s_c is the representative capitalist's propensity to save, and t_p is the (average and marginal) tax rate on profits.

Some authors have argued that the assumption of a balanced budget would be essential for such a theorem to hold in an economy with government activity. Fleck and Domenghino (1987, p. 29) reported that "Pasinetti's Paradox (dis-)solves itself in a system with government activity: The workers' propensity to save does remain a determining factor of the long-term distribution of income as well as a determinant of the long-term rate of profit."

A number of articles defending the opposite viewpoint—that the Cambridge theorem would be valid even with long-term unbalanced budgets—have been published since then: Pasinetti (1989a, 1989b), Dalziel (1989), Denicolò and Matteuzzi (1990), Araujo (1992), and Panico (1997). Following these lines of investigation, in this paper we study the relevance of the Cambridge equation in the presence of a government sector when the assumption of fixed savings is relaxed. Consumers are allowed to choose how much to consume and to save in order to add to the capital stock to provide consumption in the future at each moment of time in an infinite time horizon. As a result the marginal propensities to save are made endogenous. The neoclassical representative agent framework is adapted to include some traditional features of Kaldor-Pasinetti's models, such as different social classes. This follows the tradition pioneered by Samuelson and Modigliani (1966) to deal with Kaldor-Pasinetti models in a neoclassical framework; other examples are Darity (1981) and Faria and Teixeira (1999).

The main result of the paper is that the Cambridge equation, or Pasinetti paradox, is consistent with the model; however, the rate of profit is not determined by it. In this vein we provide micro-foundations for the two-class growth model of capital accumulation and income distribution. As Samuelson and Modigliani (1966) have argued, the Cambridge equation applies to any system capable of a golden-age growth path, which is the case of our model. However, due to its intertemporal structure the rate of profit is determined by the rate of time preference. This is a standard result in the neoclassical Ramsey-type models. Consequently, the Pasinetti paradox is no longer a paradox.

In our model the Cambridge equation provides the condition for the determination of the capitalists' marginal propensity to save. The relevance of the marginal productivity of capital is that it provides the necessary condition for the determination of the optimum quantity of capital in the economy. In addition, it is important to notice that the Cambridge equation still holds true independent of the marginal productivity of capital or any other parameter related to the production function (see Pasinetti, 1966). The equilibrium distribution of income between wages and profits, as stated by the Cambridge Theorem, is not affected by the occurrence of deficits or surpluses.

2. The Model

Let us assume at first that the economy consists of households and firms. Households are divided in two types of classes of income receivers: the capitalists, denoted Z , whose sole source of income is earnings from capital, and the remaining households, called workers, composed of mixed-income receivers and denoted N . Workers correspond to a fixed share b of the total population L . Thus we have:

$$L = Z + N \quad (2)$$

$$N = bL, \quad 0 < b < 1. \quad (3)$$

The population is assumed to increase at an exogenous rate n :

$$\frac{\dot{L}}{L} = n \Rightarrow L = e^{nt} \quad (4)$$

From Equations (2)–(4) it follows that the each social class grows at the same rate n as well. Therefore at each point in time total population is known and given.

Total real output Y is produced by labour N and total physical capital K . Total K is split into two parts: the capital owned by the capitalist class, denoted K_c , and the capital owned by workers, denoted K_w :

$$Y = F(K, N) \quad (5)$$

$$K = K_c + K_w, \quad (6)$$

where F is a well-behaved constant-returns-to-scale production function. Let us consider that $K_w = aK$, $0 < a < 1$, where a is the share of private capital owned by workers. We can normalise the variables by the total population so as to write them in per-capita terms:

$$y = \frac{Y}{L} = F(k, b) = F(k_c + k_w, b), \quad (7)$$

where the lower case letters denote the variables in per-capita units.

There are many identical competitive firms that take the rental prices of labour and capital as given to produce output. There are two factor markets, the labour market and the market for capital services. The rental price of labour, the wage, is denoted w . The rental price of capital, the interest rate or profit rate, is denoted r . Profit maximisation implies that:

$$r = F_k(k, b) \quad (8)$$

$$bw = F(k, b) - rk, \quad (9)$$

where $F_k(k, b)$ denotes the marginal productivity of capital. In per-capita terms we have:

$$y = bw + r(k_w + k_c). \quad (10)$$

This model follows Faria (2001) and assumes the neoclassical representative agent approach with Pasinettian features. Accordingly, there is a representative capitalist and a representative worker. The representative capitalist maximises his utility function over time constrained by his dynamic budget constraint:

$$\max_{c_c} \int_0^{\infty} V(c_c) e^{-(\theta-n)t} dt \quad (11)$$

$$\text{s.t.} \quad \dot{k}_c = rk_c - c_c - nk_c, \quad (12)$$

where the parameter θ is the rate of time preference, assumed to be strictly positive, and $V(\cdot)$ is the instantaneous utility function of the capitalist. It is a concave increasing function of the total consumption of the capitalists, which is denoted c_c and defined as:

$$c_c = \frac{c_c}{L} = \frac{c_c}{Z} \frac{Z}{L} = (1-b) \frac{c_c}{Z}. \quad (13)$$

Therefore the representative capitalist allocates his savings from his net income in capital. The relevant first order conditions imply that in steady state:

$$r = \theta \quad (14)$$

and

$$c_c = (r-n)k_c. \quad (15)$$

Considering that s_c is the capitalists' marginal propensity to save, it follows that:

$$s_c = \frac{n}{r}. \quad (16)$$

In the same vein, the problem of the representative worker is given by:

$$\max_{c_w} \int_0^{\infty} U(c_w) e^{-(\theta-n)t} dt \quad (17)$$

$$\text{s.t.} \quad \dot{k}_w = rk_w + bw - c_w - nk_w, \quad (18)$$

where $U(\cdot)$ is a concave increasing function of the workers' total consumption, denoted c_w and defined as:

$$c_w = \frac{c_w}{L} = \frac{c_w}{N} \frac{N}{L} = b \frac{c_w}{N}. \quad (19)$$

Notice that the representative worker supplies labour inelastically. He allocates his savings from his income in capital and labour. In steady state the relevant first order conditions imply that:

$$r = \theta \tag{20}$$

and

$$c_w = (r - n)k_w + bw . \tag{21}$$

Considering that s_w denotes the workers' marginal propensity to save, it follows that:

$$s_w = \frac{nk_w}{rk_w + bw} . \tag{22}$$

Expression (16) may be rewritten as $r = n/s_c$, which is one of versions of the Cambridge equation. But in the present treatment it does not determine the rate of profit of the economy since this rate is determined by Equation (14). Hence the role of the Cambridge equation is to determine, given the long-run equilibrium rate of profit, the equilibrium value of the capitalist's marginal propensity to save.

Samuelson and Modigliani (1966) show that if the workers' marginal saving propensity is high enough, this class would end up doing all accumulation and the capitalist's share of output would vanish to zero. This result is known as the dual equilibrium, also called the "euthanasia" of the capitalists. However, in the present model, this outcome is not possible. Notice that from Equations (16) and (17) we have $0 < s_w < s_c$. In the next section, we verify the validity of these results in the presence of government activity.

3. The Government Sector

The introduction of the government sector in the Kaldor-Pasinetti models of growth and distribution may be accomplished using alternative formulations in terms of whether the budget is balanced or not, whether the government accumulates capital or not, patterns of spending and revenue, etc. The approach adopted here assumes that the government may own capital goods. The existence of public capital is a reasonable assumption when the government is allowed to save (or dissave) (see Araujo, 1992, p. 221). The government consumption and investment is financed not only by taxes but also by government net profits from the public capital K_g . In this case Equations (6), (7), and (10) must be replaced by Equations (23), (24), and (25) as follows:

$$K = K_c + K_w + K_g \tag{23}$$

$$y \equiv \frac{Y}{L} = F(k, b) = F(K_c + K_w + K_g, b) \tag{24}$$

$$y = bw + r(K_w + K_c + K_g) . \tag{25}$$

Let us consider that $K_w = a(K - K_g)$, $0 < a < 1$, where a is the share of private capital owned by workers. The government's per capita demand for resources is exogenous and, consequently, does not directly affect the marginal utility of consumption of the representative capitalist and worker. The government budget constraint is the following:

$$g + \dot{k}_g + nk_g = \tau + (1 - t_p)rk_g, \quad (26)$$

where g stands for the government expenditures and total per capita taxes are given by:

$$\tau = t_w bw + t_p rk. \quad (27)$$

Notice that the government also taxes its own profits. Furthermore, s_g stands for the government's marginal propensity to save and is considered as exogenously given (in contrast with the marginal propensity to save of capitalists and workers, which are endogenous), it follows that:

$$\dot{k}_g = s_g [t_w(y - rk) + t_p rk] + s_g(1 - t_p)rk_g - nk_g. \quad (28)$$

In the steady state, government spending equals government revenue:

$$\dot{k}_g = 0 \Rightarrow g = \tau + (1 - t_p)rk_g - nk_g. \quad (29)$$

Therefore, from Equation (26) the value of k_g is determined as follows:

$$k_g = \left[\frac{t_w(y - rk) + t_p rk}{n - s_g(1 - t_p)r} \right] s_g. \quad (30)$$

It is important to stress the fact that in the steady state, the government budget constraint (given by Equation (28)) is balanced; that is, government expenditures are equal government's net income. This fact excludes the possibility of permanent deficits or surpluses since they are not consistent with the steady state analysis in a general equilibrium model. In this context the problem to be solved by the representative capitalist is the following:

$$\max_{c_c} \int_0^{\infty} V(c_c) e^{-(\theta-n)t} dt \quad (31)$$

$$\text{s.t.} \quad \dot{k}_c = (1 - t_p)rk_c - c - nk_c. \quad (32)$$

Concerning the capitalist's budget constraint given by Equation (32), t_p is the profit tax rate. Therefore the representative capitalist allocates his savings from his net income in capital. The relevant first order conditions (see the Appendix) imply that in steady state:

$$r = \frac{\theta}{(1-t_p)} \quad (33)$$

and

$$c = [(1-t_p)r - n]k_c. \quad (34)$$

Considering that s_c is the capitalists' marginal propensity to save, it follows that:

$$s_c = \frac{n}{(1-t_p)r}. \quad (35)$$

The representative worker solves now the following problem:

$$\max_{c_w} \int_0^{\infty} U(c_w) e^{-(\theta-n)t} dt \quad (36)$$

$$\text{s.t.} \quad \dot{k}_w = (1-t_p)rk_w + (1-t_w)bw - c_w - nk_w. \quad (37)$$

Considering that t_w stands for the wage tax rate, Equation (37) is the worker's budget constraint, which is analogous to the capitalist's. The representative worker allocates his savings from his net income in capital. In steady state the relevant first order conditions (see the Appendix) imply that:

$$r = \frac{\theta}{(1-t_p)} \quad (38)$$

and

$$c_w = [(1-t_p)r - n]k_w + (1-t_w)bw. \quad (39)$$

The workers' marginal propensity to save s_w is given by:

$$s_w = \frac{nk_w}{[(1-t_p)rk_w + (1-t_w)bw]}. \quad (40)$$

Analogous to the previous case, Equation (35) may be rewritten as: $r = n/(1-t_p)s_c$, which is the same version obtained by Steedman (1972) and Denicolò and Matteuzzi (1990). However, as shown in the next section, it does not determine the rate of profit since it is determined by Equation (33).

4. General Equilibrium and Comparative Static Analysis

In order to analyse this general equilibrium model, let us consider the steady state solution of the model, which is given by the following equations:

$$r^* = \frac{\theta}{(1-t_p)} \quad (41)$$

$$r = F_k(k^*, b) \quad (42)$$

$$bw^* = F(k, b) - rk \quad (43)$$

$$y^* = F(k, b) \quad (44)$$

$$k_g^* = \left[\frac{t_w(y - rk) + t_p rk}{n - s_g(1-t_p)r} \right] s_g \quad (45)$$

$$k_w^* = a(k - k_g) \quad (46)$$

$$k - k_g = k_c^* + k_w \quad (47)$$

$$c_c^* = [(1-t_p)r - n]k_c \quad (48)$$

$$c_w^* = [(1-t_p)r - n]k_w + (1-t_w)bw. \quad (49)$$

There are nine endogenous variables ($r, k, w, y, k_g, k_w, k_c, c_c, c_w$) for nine equations. The steady state solutions are arranged in such a way to highlight the recursiveness of the model, and an asterisk signals the determination of the steady state values of the endogenous variables.

The first and main result of this paper is that the long-run equilibrium profit rate r^* is determined by Equation (41) and is equal to the rate of time preference corrected by the profit tax rate. As for the remaining endogenous variables, notice that given the value of r^* , Equation (42) determines the steady state value of physical capital stock per capita k^* . It is in here that the marginal productivity of capital plays a role in the model. Given k^* , Equations (43), (44), (45), (46), and (47) determine the equilibrium values of w^* , y^* , k_g^* , k_w^* , and k_c^* respectively. Given k_c^* , Equation (48) determines the optimal value of c_c^* while Equation (49) determines the equilibrium value of c_w^* given k_w^* .

The second important result of this paper is that the equilibrium distribution of income between wages and profits is not affected by the occurrence of government deficits or surpluses. The role of the Cambridge equation in this model raises the third result of this paper. In fact, the Cambridge equation is a solution of this model; however, it no longer determines the equilibrium rate of profit. It is given by Equation (35) rewritten as $r = n/(1-t_p)s_c$, and it determines the capitalist's marginal propensity to save, which is endogenous in this model. In addition, Equation (36) confirms what was reported by Baranzini (1991, p. 115): "The optimal equilibrium interest rate r^* does not depend on the form of the production function nor on the value of the capital/labour ratio.... In a certain sense the simplicity of the Meade-Samuelson and Modigliani and Kaldor-Pasinetti theorems is repeated. Additionally the fact that r^* does not depend on the form of the production function seems to confirm the validity of Kaldor-Pasinetti theorem."

It is important to stress the macroeconomic consistency of the model. Notice that from the steady state solution we have:

$$y = bw + r(K_w + K_c + K_g) = c_c + c_w + n(K_w + K_c + K_g) + g . \quad (50)$$

The comparative statics of the model are quite simple. It is easy to see that a rise in the profit tax rate leads to a rise in the interest rate, $dr^*/dt_p = \theta/(1-t_p)^2 > 0$, which decreases the optimal amount of aggregate capital, k^* , causing the wages, w^* , and output, y^* , to decrease as well. As a consequence capitalists', c_c^* , and workers', c_w^* , consumption are also negatively affected by the increase in the profit tax rate. In line with the Cambridge equation, the tax rate on wages does not affect the rate of profit, capital accumulation, and output in this economy.

These results were derived under the hypothesis that both the representative capitalist and worker have the same rate of intertemporal discount. This assumption is a reasonable one since the only difference between the two classes is that the representative worker is a mixed income receiver while the income of the representative capitalist accrues only from earnings of capital. However, even if this assumption is dropped the above results remain valid; that is, if we assume that the rate of intertemporal discount of workers is higher than that of capitalists—which seems the plausible possibility—the Cambridge equation is a solution of the model but it does not determine the rate of profit. The marginal propensities to save remain endogenous and the inequality $0 < s_w < s_c$ continues to hold. This last fact excludes the possibility of the dual result.

5. Conclusion

One of the main lines of research of post-Keynesian economists is to provide some micro-foundations for the two-class growth model of capital accumulation and income distribution. As pointed out by Baranzini (1991, p. 107), "... the idea of the introduction of some micro-foundations (as the life-cycle hypothesis) in the traditional two-class model of distribution is somewhat recent in the literature." Following this line of investigation, this paper analyses the relevance of the Cambridge equation in the presence of a government sector, when the assumption of fixed savings is relaxed.

We consider an intertemporal representative agent model with Pasinettian features. That is, the model presents two social classes, capitalists and workers, and is able to analyse the impact of government fiscal policies on growth and income distribution. In this vein we have verified that the equilibrium distribution of income between wages and profits, as stated by the Cambridge theorem, is not affected by the occurrence of deficits or surpluses as claimed by Fleck and Domenghino (1987, 1990). These results are consistent with the conclusions of Denicolò and Matteuzzi (1990) and Araujo (1992).

The model also provides other interesting results. The most important is that the rate of profit is not determined by the Cambridge equation. This equation determines the equilibrium value of the capitalist's marginal propensity to save. It is worth noticing that the Cambridge equation is independent of the marginal product of capital and any other parameter of the production function. Finally, it is shown

that only taxation on profits affects the profit rate and, as a consequence, capital accumulation, wages, and output. Taxation on wages does not affect the profit rate, capital accumulation, or output.

Appendix

Solution for the problem of the representative capitalist

The Hamiltonian function for problem (31)–(32) may be written as:

$$H = V(c_c) + \mu [(1-t_p)rk_c - c_c - nk_c].$$

Considering that μ is the co-state variable associated with k_c , the first order conditions may be expressed as:

$$\begin{aligned} H_{c_c} = 0 &\Rightarrow V'(c_c) = \mu \\ \dot{\mu} - (\theta - n)\mu &= -H_{k_c} \\ \Rightarrow \dot{\mu} - (\theta - n)\mu &= -\mu[(1-t_p)r + n] \\ \Rightarrow \frac{\dot{\mu}}{\mu} &= [\theta - (1-t_p)r] \end{aligned}$$

plus the transversality condition. In steady state we obtain:

$$\frac{\dot{\mu}}{\mu} = 0 \Rightarrow \theta - (1-t_p)r = 0 \Rightarrow \theta = (1-t_p)r \Rightarrow r = \frac{\theta}{1-t_p},$$

which is Equation (33) and:

$$\frac{\dot{k}_c}{k_c} = 0 \Rightarrow c_c = [(1-t_p)r - n]k_c,$$

which is Equation (34). Considering that S_c is the total savings of the capitalists and s_c is the capitalists' marginal propensity to save, it follows that:

$$\begin{aligned} \frac{s_c Y_c}{L} &= \frac{Y_c}{L} - \frac{C}{L} \Rightarrow s_c(1-t_p)rk_c = (1-t_p)rk_c - c_c \\ \Rightarrow s_c(1-t_p)rk_c &= (1-t_p)rk_c - [(1-t_p)r - n]k_c \\ \Rightarrow s_c(1-t_p)r &= n \\ \Rightarrow s_c &= \frac{n}{(1-t_p)r}, \end{aligned}$$

which is Equation (35).

Solution for the problem of the representative worker

The Hamiltonian function for problem (36)–(37) may be written as:

$$H = U(c_w) + \lambda [(1-t_p)rk_w + (1-t_w)bw - c_w - nk_w].$$

Considering that λ is the co-state variable associated with k_w , the first order conditions may expressed as:

$$\begin{aligned} H_{c_w} = 0 &\Rightarrow U'(c_w) = \lambda \\ \dot{\lambda} - (\theta - n)\lambda &= -H_{k_w} \Rightarrow \dot{\lambda} - (\theta - n)\lambda = -\lambda[(1-t_p)r - n] \\ \Rightarrow \frac{\dot{\lambda}}{\lambda} &= \theta - (1-t_p)r \end{aligned}$$

plus the transversality condition. In steady state we obtain:

$$\frac{\dot{\lambda}}{\lambda} = 0 \Rightarrow \theta - (1-t_p)r = 0 \Rightarrow \theta = (1-t_p)r \Rightarrow r = \frac{\theta}{1-t_p},$$

which is Equation (38) and:

$$\frac{\dot{k}_w}{k_w} = 0 \Rightarrow c_w = [(1-t_p)r - n]k_w + (1-t_w)bw,$$

which corresponds to Equation (39).

Considering that S_w and s_w denote workers' total savings and marginal propensity to save, it follows that:

$$\begin{aligned} \frac{s_w Y_w}{L} &= \frac{Y_w}{L} - \frac{C_w}{L} \Rightarrow s_w [(1-t_p)rk_w + (1-t_w)bw] = (1-t_p)rk_w + (1-t_w)bw - c_w \\ \Rightarrow s_w [(1-t_p)rk_w + (1-t_w)bw] &= (1-t_p)rk_w + (1-t_w)bw - [(1-t_p)r - n]k_w - (1-t_w)bw \\ &\Rightarrow s_w [(1-t_p)rk_w + (1-t_w)bw] = nk_w \\ \Rightarrow s_w &= \frac{nk_w}{[(1-t_p)rk_w + (1-t_w)bw]} \end{aligned}$$

which is Equation (40).

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