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# Tradable Emission Permits Regulations: The Role of Product Differentiation

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**Abstract**

This paper examines the role of product differentiation within the model of Sartzetakis (1997, 2004) and shows that consumer surplus may be reduced under a tradable emission permits system rather than a command and control system when there is a high degree of product differentiation or less competition between two firms. We also investigate comparative static effects of the degree of product differentiation on equilibrium output and abatement levels under the two regulatory regimes.

*Key words*: command and control system; product differentiation; tradable emission permits system

*JEL classification*: L1; L5; Q2

## 1. Introduction

In light of the increasing importance of environmental regulation, the widespread acceptance of a tradable emission permits (TEP) system generates an interesting debate among policy makers on the efficiency of TEP regulation and its comparison with the command and control (CAC) regulation. Many researchers are of the opinion that governments can promote social welfare by implementing a TEP system, which minimizes abatement costs when they differ between firms. However, even under a competitive permits trading market, Borenstein (1988) and Malueg (1990), for example, indicated the possibility that a TEP system may reduce both consumer surplus and social welfare when the product market is not perfectly competitive. In addition, Malik (1990, 2002), Keeler (1991), and Stranlund and Dhanda (1999) cast doubt on the efficiency properties of the TEP system when firms may be noncompliant.

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In contrast, Sartzetakis (1997) considered a homogeneous Cournot duopoly model under the assumption of constant and equal costs of production between firms and showed that TEP regulation can increase not only consumer surplus but also social welfare under certain conditions. However, Sartzetakis (2004) demonstrated that if firms differ in both production and abatement technologies, competition in the emission permits market cannot always assure efficiency when the product market is duopolistic.

The present paper extends the analysis of Sartzetakis (1997, 2004) by considering the role of product differentiation and shows that consumer surplus might be reduced under a TEP rather than a CAC system when there is a high degree of product differentiation or less competition between two firms. This result implies that the degree of product differentiation is an important factor that needs to be taken into account in the design of TEP regulations. We also investigate comparative static effects of the degree of product differentiation on equilibrium output and abatement levels under the two regulatory regimes considered.

## 2. The Model

We examine the Cournot duopoly model where firms sell differentiated goods and compete by setting quantities. Extending recent work by Sartzetakis (1997, 2004), we analyze in detail the effect of product differentiation on consumer surplus and social welfare. In line with Dixit (1979) and Singh and Vives (1984), we postulate that there is a representative consumer whose preferences for consumption of the two goods are described by

$$U = a(q_1 + q_2) - \frac{q_1^2 + 2\gamma q_1 q_2 + q_2^2}{2}, \tag{1}$$

where *a* (>0) is a constant measure of market size, $q_i$ is the output of the firm *i* (*i* = 1, 2), and $\gamma \in (0,1)$ captures the degree of product differentiation: the higher $\gamma$, the lower the degree of product differentiation. Thus, a low $\gamma$ corresponds to a situation of rather limited competition and a higher $\gamma$ captures intensified competition.

The extreme cases are illustrative. As $\gamma$ approaches 0, the two firms are effectively local monopolists. As $\gamma$ approaches 1, the two goods are increasingly homogeneous. In fact, $\gamma = 1$ corresponds to the duopoly with homogeneous products case of Sartzetakis (1997, 2004), where two products are perfect substitutes. This implies that a representative consumer has a linear indifference curve in his preferences space. Therefore, the preferences with $\gamma \in (0,1)$ that are defined in (1) represent consumers with convex preferences.

This specification results in a linear demand structure with

$$p_i = a - q_i - \gamma q_j, \quad i, j = 1,2, \quad i \neq j, \tag{2}$$

where $p_i$ is the price of good $i$.

Production generates an emission $E_i$ of a pollutant. Following the assumptions of Malueg (1990) and Sartzetakis (1997, 2004), the emission level is dependent on both the production level $q_i$ and the emission abatement activity level $z_i$. Specifically, $E_i = (\rho - z_i)q_i$ where $\rho$ ($> z_i > 0$) is the constant emission rate per unit of output, which is the identical for the two firms.

On the cost side, like Sartzetakis (1997) we assume that total costs are increasing in output and abatement: $C_i(q_i, z_i) = e_i z_i^2 q_i^2$, where $e_i$ represents the technological difference between the two firms. In other words, with the same $q_i$ and $z_i$, the larger $e_i$, the greater the costs of emission reduction. We assume that $e_1 > e_2 > 0$ so that firm 1 (2) has a less (more) efficient emission reduction technology.

Finally, it is assumed that the government plans to reduce emission levels to $\overline{E}$, which is determined in the political arena, perhaps through international agreements on emission reductions or folding under the pressure of special interest groups. The problem that environmental policy makers consider is to choose between a CAC system and a TEP system to control current pollution levels.

### 3. Command and Control Regulation

Under a CAC system, regulators impose a non-tradable emission quota on firm *i* ($\overline{E}_i$) and on the industry ($\overline{E} = \overline{E}_1 + \overline{E}_2$). Here, we consider the symmetric case where $\overline{E}_1 = \overline{E}_2$. Firm $i$ chooses output and abatement levels in order to maximize its profits $\pi_i = (a - q_i - \gamma q_j)q_i - e_i z_i^2 q_i^2$ subject to the emission constraint $\overline{E}_i \geq (\rho - z_i)q_i$. That is, the optimization problem is:

$$\max_{\{q_i>0,\, z_i>0\}} L_i = (a - q_i - \gamma q_j)q_i - e_i z_i^2 q_i^2 + \lambda_i\left(\overline{E}_i - (\rho - z_i)q_i\right). \tag{3}$$

Then, the first order Kuhn-Tucker conditions for firms are as follows:

$$\frac{\partial L_i}{\partial q_i} = a - 2q_i - \gamma q_j - 2e_i q_i z_i^2 - \lambda_i(\rho - z_i) = 0, \tag{4}$$

$$\frac{\partial L_i}{\partial z_i} = -2e_i q_i^2 z_i + \lambda_i q_i = 0, \tag{5}$$

$$\frac{\partial L_i}{\partial \lambda_i} = \overline{E}_i - (\rho - z_i)q_i \geq 0, \quad \lambda \geq 0, \quad \lambda \frac{\partial L_i}{\partial \lambda_i} = 0. \tag{6}$$

From (5), the implicit price for the emission quota given $q_i$ is equal to the marginal cost of emission abatement, i.e., $\lambda_i q_i = 2e_i z_i q_i^2 = \partial C_i / \partial z_i > 0$. Thus, the emission constraint in (6) is binding, i.e., $\overline{E}_i - (\rho - z_i)q_i = 0$. Now, substituting $\lambda_i = 2e_i z_i q_i$ into (4), we obtain two output response functions for each firm. Solving these two simultaneous equations gives the following optimal output and abatement levels for firm *i* at equilibrium:

$$q_i^c = \frac{a(2-\gamma) - \rho(2\lambda_i - \gamma\lambda_j)}{4-\gamma^2}, \tag{7}$$

$$z_i^c = \frac{\lambda_i}{2e_i q_i^c} = \frac{\lambda_i(4-\gamma^2)}{2e_i\left(a(2-\gamma) - \rho(2\lambda_i - \gamma\lambda_j)\right)}. \tag{8}$$

To continue our analysis, we calculate the sum of two products and outputs difference as follows:

$$Q^c \equiv q_1^c + q_2^c = \frac{2a - \rho(\lambda_1 + \lambda_2)}{2+\gamma}, \tag{9}$$

$$q_1^c - q_2^c = \frac{\rho(\lambda_2 - \lambda_1)}{2-\gamma}. \tag{10}$$

Note that the output difference depends on the implicit price difference. In particular, the firm that has a lower implicit price for emissions produces more output in equilibrium; i.e., $\lambda_i > \lambda_j$ implies $q_j^c > q_i^c$. Note also that from the binding emission constraint in (6), $z_i^c$ is proportional to $q_i^c$. This also implies that $\lambda_i > \lambda_j$ yields $z_j^c > z_i^c$. Thus, we have the following lemma (see Appendix A for the proof).

***Lemma 1*****:** If $e_1 > e_2$, then $\lambda_1 > \lambda_2$.

***Proposition 1*****:** Under a CAC regulation system, a more (less) efficient firm has higher (lower) output levels and higher (lower) emission abatement efforts in equilibrium. (In our notation: $q_2^c > q_1^c > 0$ and $z_2^c > z_1^c > 0$.)

The proposition shows that a less (more) efficient firm has a higher (lower) implicit price. Thus, this firm produces lower (higher) outputs, leading to lower (higher) abatements. However, this result depends on both the degree of product differentiation and the emission quota. For example, from (A1) in Appendix A, we see that if the regulator reduces the emission quota for each firm, its price rises: $\partial\lambda_i / \partial\overline{E}_i < 0$.

Now we consider comparative static of the degree of product differentiation. Using the comparative statics of Edlin and Shannon (1998), we know that the optimal output level $q_i^c$ is increasing in the degree of product differentiation if $(\partial\pi/\partial q_i)/|\partial\pi/\partial z_i|$ is decreasing in $\gamma$ at $(q_i^c, z_i^c)$. Then, we obtain

$$\frac{\partial\pi/\partial q_i}{|\partial\pi/\partial z_i|}(q_i^c, z_i^c) = \frac{a - q_i^c - \gamma q_j^c - q_i^c - 2e_i z_i^c 2q_i^c}{|-2e_i q_i^c 2z_i^c|},$$

which is decreasing in $\gamma$. Therefore, $\partial q_i^c/\partial\gamma < 0$, implying $\partial Q^c/\partial\gamma < 0$. This implies that as the degree of product differentiation decreases (i.e., $\gamma$ increases or, equivalently, competition increases), equilibrium output decreases. In addition, from the same procedure of obtaining $\partial q_i^c/\partial\gamma$, we obtain $\partial z_i^c/\partial\gamma < 0$. This implies that as the degree of product differentiation decreases, the equilibrium abatement effort decreases. Finally, from the equilibrium condition in (5), we easily derive that the implicit price $\lambda_i$ increases (decreases) as long as both $q_i^c$ and $z_i^c$ increase (decrease). Therefore, if the degree of product differentiation decreases or the degree of competition increases, the implicit price also decreases: $\partial\lambda_i/\partial\gamma < 0$ . The intuition is as follows: since the equilibrium output for each firm decreases as $\gamma$ increases, the firms put a lower value on the emission quotas relative to the case of a lower $\gamma$ , i.e., the implicit price for the emission quota decreases.

However, the effects of the degree of product differentiation on each firm differ. For example, from (A2) in Appendix A, we obtain that $\partial(\lambda_1 - \lambda_2)/\partial\gamma < 0$ when $\gamma \in (0,1)$ , i.e., $\partial\lambda_1/\partial\gamma < \partial\lambda_2/\partial\gamma < 0$ . Therefore, as the degree of product differentiation decreases, the implicit price of a less efficient firm decreases faster than that of a more efficient firm.

## 4. Tradable Emission Permits Regulation

Under a TEP system, regulators assign an emission quota $\overline{E}_i$ to each firm and allow it to trade emission permits at the market price. Following Sartzetakis (1997, 2004), we assume that the market price of permits is determined by the market clearing price. Thus, each firm behaves as a price taker in the emission market. Therefore, if we define the net demand of firm *i* as $D_i = \overline{E}_i - (\rho - z_i)q_i$, total net demand of emission permits is zero at the market equilibrium $D_1 + D_2 = 0$ .

Each firm maximizes the following profit function under a TEP system:

$$\max_{\{q_i>0, z_i>0\}} \quad \pi_i^T = \left(a - q_i - \gamma q_j\right)q_i - e_i q_i^2 z_i^2 - t\left(\left(\rho - z_i\right)q_i - \overline{E}_i\right), \tag{11}$$

where *t* is the market clearing price of permits. The first-order conditions are

$$\frac{\partial \pi_i^T}{\partial q_i} = a - 2q_i - \gamma q_j - 2e_i q_i z_i^2 - t(\rho - z_i)q_i = 0 , \tag{12}$$

$$\frac{\partial \pi_i^T}{\partial z_i} = -2e_i q_i^2 z_i + tq_i = 0 . \tag{13}$$

From (13), we see that the market price for emission permits given $q_i$ is equal to the marginal cost of the emission abatement, i.e., $tq_i = 2e_i q_i^2 z_i = \partial C_i/\partial z_i > 0$ . Substituting this into (12) and solving two output response functions for each firm simultaneously, we can obtain the following optimal output and abatement levels for firm *i* at equilibrium:

$$q_i^T = \frac{a - t\rho}{2 + \gamma} , \tag{14}$$

$$z_i^T = \frac{t}{2e_i q_i^T} = \frac{t(2+\gamma)}{2e_i(a - t\rho)} . \tag{15}$$

From (14), we see that each firm has the same output levels at equilibrium. This is due to the symmetric market assumption characterized in (2). From $q_1^T = q_2^T$, we also have $e_1 z_1^T = e_2 z_2^T$ at equilibrium from (15). Therefore, $z_2^T > z_1^T$ when $e_1 > e_2$. This indicates that a more (less) efficient firm makes higher (lower) abatement efforts to reduce emissions. Finally, for comparison, we observe that

$$Q^T \equiv q_1^T + q_2^T = \frac{2(a - \rho t)}{2 + \gamma}. \tag{16}$$

***Proposition 2*:** Under a TEP system, both firms have the same output levels and a more (less) efficient firm makes higher (lower) abatement efforts in equilibrium. (In our notation: $q_2^T = q_1^T > 0$ and $z_2^T > z_1^T > 0$.)

Proposition 2 also depends on the degree of product differentiation. For comparative statics of the degree of product differentiation, consider the equilibrium characterized by (14) and (15) under a TEP system. Substituting these into the total net demand of the emission quota, where $D_1 + D_2 = \sum_{i=1}^{2} \{\overline{E}_i - (\rho - z_i^T) q_i^T\} = 0$, and rearranging yields

$$t = \frac{2e_1 e_2 (2a\rho - (2+\gamma)\overline{E})}{(2+\gamma)(e_1 + e_2) + 4\rho^2 e_1 e_2}. \tag{17}$$

One can easily check that the market price of permits is higher if the regulator reduces the emission quota for each firm, i.e., $\partial t / \partial \overline{E}_i < 0$, if the degree of product differentiation increases, or if $\gamma$ decreases, i.e., $\partial t / \partial \gamma < 0$. Therefore, a high (low) degree of competition between two firms implies a lower (higher) permits price.

From (14) and (16), we obtain that $\partial q_i^T / \partial t < 0$, which implies that $\partial Q^T / \partial t < 0$. Also, from (15), it follows that $\partial z_1^T / \partial t > \partial z_2^T / \partial t > 0$ since $e_1 > e_2$. This implies that as the permit price increases, firms have an incentive to reduce outputs and to increase abatements so as to decrease their need for permits.

Finally, in order to see the effect of product differentiation on outputs and abatements, substituting the equilibrium value of $t$ from (17) into (14) and (15) yields $\partial q_i^T / \partial \gamma < 0$ and $\partial z_i^T / \partial \gamma < 0$, respectively. Therefore, as the degree of product differentiation decreases or $\gamma$ increases, equilibrium output and abatements also decrease under a TEP system.

## 5. Comparing Policy Regimes

We first provide a comparison in terms of output and abatement levels under the two regulatory regimes. We summarize our findings in Lemma 2 (see Appendix A for the proof.)

***Lemma 2*:** If $e_1 > e_2$, then $0 < \lambda_2 < t < \lambda_1$ and $t < (\lambda_1 + \lambda_2)/2$.

Lemma 2 shows that when two firms trade permits in the emission market under a TEP system, the permit price is determined at a level lower than the implicit price of the emission quota of the less efficient firm and higher than that of the more efficient firm (i.e., $0 < \lambda_2 < t < \lambda_1$) in order to make trading permits possible. However, the permits price level is less than half of the total implicit prices levels (i.e., $t < (\lambda_1 + \lambda_2)/2$)—that is, the average marginal abatement cost of two firms. This implies that the more efficient firm can earn higher profits compared to the less efficient firm.

***Lemma 3*:** If $e_1 > e_2$, then the following relationships hold:

$$\Delta q_i \equiv q_i^T - q_i^c = \frac{\rho\big(2(\lambda_i - t) + \gamma(t - \lambda_j)\big)}{4 - \gamma^2}, \tag{18}$$

$$\Delta z_i \equiv z_i^T - z_i^c = \frac{tq_i^c - \lambda_i q_i^T}{2e_i q_i^T q_i^c}, \tag{19}$$

$$\Delta Q \equiv Q^T - Q^c = \frac{\rho(\lambda_1 + \lambda_2 - 2t)}{(2+\gamma)}. \tag{20}$$

A few remarks are in order. First, using Lemmas 2 and 3, we obtain $q_1^T > q_1^c$ and $q_2^T < q_2^c$ from (18). This implies that the less (more) efficient firm increases (decreases) its output level by buying (selling) the emission permits under a TEP system. Using this result, we observe that $z_2^T > z_2^c > z_1^c > z_1^T$ from (19). Therefore, a more (less) efficient firm increases (decreases) its abatement efforts under a TEP system. Finally, we conclude that $Q^T > Q^c$ from (20). This implies total outputs increase by shifting the regulatory regimes from a CAC system to a TEP system. This is so because the permits price is lower than the average marginal abatement cost of the two firms. Thus, the output increment of the less efficient firm overwhelms the output decrement of the more efficient firm. We summarize these results in Proposition 3.

***Proposition 3*:** Shifting regulatory policy from a CAC system to a TEP system implies that (i) the less (more) efficient firm increases (decreases) its output level, (ii) the less (more) efficient firm decreases (increases) its abatement efforts, and (iii) total outputs increase.

Using the result that total output increases under a TEP system (i.e., $Q^T > Q^c$), we conclude that if the two firms supply identical and perfectly substitutable products (i.e., $\gamma = 1$), market price decreases. Thus, consumer surplus necessarily increases under a TEP system, a result established in Sartzetakis (1997). Intuitively, this is because the consumer's indifference curve in preferences space is linear. Thus, the result that total output increases implies that consumer surplus increases.

However, from $q_1^c < q_1^T = q_2^T < q_2^c$, the effect of product differentiation on consumer surplus in each market differs when $\gamma \neq 1$. Under the preferences defined in (1), when the consumer's preferences are convex, consumer surplus might not increase even though total output increases. For example, consider the extreme case of $\gamma = 0$ where the two markets are independent. Then the consumer surplus in market 1 increases as much as $(q_1^T + q_1^c)(q_1^T - q_1^c)/2$, which is positive, while that of market 2 decreases as much as $(q_2^T + q_2^c)(q_2^T - q_2^c)/2$, which is negative. Hence, consumer surplus increases only if $q^T > q_1^c + (q_2^c - q_1^c)/(1+k)$ where $k = (q^T + q_1^c)/(q^T + q_2^c) < 1$. Thus, the sign of the increment in total consumer surplus depends on the relative size of equilibrium outputs in each market.

Next, we provide a comparison in terms of the consumer surplus and social welfare under the two regulatory regimes. First, the consumer surplus is defined as

$$CS = U - p_1 q_1 - p_2 q_2 = \frac{q_1^2 + 2\gamma q_1 q_2 + q_2^2}{2},$$

where $U$ is the consumer's utility defined in (1). Then, the difference in consumer surplus between the two regulatory regimes is

$$\Delta CS \equiv CS^T - CS^c = \frac{(Q^T - Q^c)(Q^T + Q^c)}{2} - (1-\gamma)(q_1^T q_2^T - q_1^c q_2^c), \tag{21}$$

Notice that both terms in (21) are positive. Therefore, the sign of $\Delta CS$ in general is ambiguous. (From the Proposition 3, we have $q_1^T + q_2^T > q_1^c + q_2^c$. By squaring both sides of this inequality, we have $4q_1^T q_2^T > (q_1^c)^2 + (q_2^c)^2 + 2q_1^c q_2^c$ since $q_1^T = q_2^T$. From this we conclude that the second term in (21) is positive since $q_1^T q_2^T - q_1^c q_2^c > 0$.) This implies that the difference in consumer surplus in (21) depends on the degree of product differentiation.

In particular, we have the following relationship:

$$\Delta CS > 0 \quad \text{if} \quad \gamma > 1 - (Q^T - Q^c)(Q^T + Q^c)/2(q_1^T q_2^T - q_1^c q_2^c), \tag{22}$$

Notice that $(Q^T - Q^c)(Q^T + Q^c)/2(q_1^T q_2^T - q_1^c q_2^c) > 0$, which implies that there is a threshold for $\gamma$ below which consumer surplus decreases under a TEP system.

In contrast, the difference in social welfare is the difference between the difference of consumer utility in (1) and the cost changes:

$$\Delta SW \equiv SW^T - SW^c$$
$$= (Q^T - Q^c)\left(a - \frac{Q^T + Q^c}{2}\right) + (1-\gamma)(q_1^T q_2^T - q_1^c q_2^c) - \left[\sum_{i=1}^{2} e_i (q_i^T z_i^T)^2 - \sum_{i=1}^{2} e_i (q_i^c z_i^c)^2\right], \tag{23}$$

where $\gamma = 1$. Notice that the environmental damage is not included in (23) since the emission level, $\overline{E}$, is the same under two different policy regimes. Notice also that the first term and the second term are positive while the last term of $[\sum_{i=1}^2 e_i (q_i^T z_i^T)^2 - \sum_{i=1}^2 e_i (q_i^c z_i^c)^2]$ might be positive since $z_2^T > z_2^c > z_1^c > z_1^T$ and $q_1^c < q_1^T = q_2^T < q_2^c$. Thus, the sign of $\Delta SW$ in general is ambiguous. Therefore, the difference in social welfare in (23) also depends on the degree of product differentiation.

We have that

$$\Delta SW > 0 \quad \text{if} \quad \gamma < 1 + \frac{\left(Q^T - Q^c\right)\left(a - \left(Q^T + Q^c\right)/2\right) - \left[\sum_{i=1}^{2} e_i \left(q_i^T z_i^T\right)^2 - \sum_{i=1}^{2} e_i \left(q_i^c z_i^c\right)^2\right]}{\left(q_1^T q_2^T - q_1^c q_2^c\right)} \,. \tag{24}$$

Notice that the change in social welfare always increases under a TEP system by as much as $(Q^T - Q^c)(a - (Q^T + Q^c)/2) - [\sum_{i=1}^{2} e_i (q_i^T z_i^T)^2 - \sum_{i=1}^{2} e_i (q_i^c z_i^c)^2] > 0$. Therefore, social welfare increases if the change in total output outweighs the change in total abatement costs.

As pointed out in Sartzetakis (2004), the intuition of this result is easier to understand in the context of the second best problem, where the environmental policy maker chooses between a CAC system and a TEP system. On the one hand, due to imperfect competition in a differentiated product market which lessens competition between the firms, the less efficient firm produces higher output while the more efficient firm produces lower output under a TEP system (i.e., $q_1^c < q_1^T = q_2^T < q_2^c$). On the other hand, due to non-tradable emission quotas, the less efficient firm devotes more resources to abatement activity while the more efficient firm takes devotes fewer resources to abatement activity under a CAC system (i.e., $z_2^T > z_2^c > z_1^c > z_1^T$). Thus, allowing firms to transfer emission permits through a competitive market corrects the abatement misallocation problem but could aggravate the production misallocation problem in a differentiated product market. Therefore, the change in social welfare depends not only on production cost differences as in Sartzetakis (2004) but also on the degree of product differentiation.

## 6. Example

In this section, we examine a simple example for further comparative analysis. Specifically, we consider a numerical example with $a$ = 25, $e_1$ = 9, $e_2$ = 3, $\rho$ = 1, and $\overline{E}_i$ =3/2. We obtain results for various values of $\gamma$ as reported in Tables 1 through 3.

First, from Table 1, we observe the results indicated in Proposition 1 under a CAC system. In particular, we have $\partial\lambda_i/\partial\gamma < 0$, $\partial q_i^c/\partial\gamma < 0$, and $\partial z_i^c/\partial\gamma < 0$. As a consequence, as the degree of product differentiation decreases or competition increases, profit decreases and consumer surplus increases but social surplus decreases: $\partial\pi_i^c/\partial\gamma < 0$, $\partial\Pi^c/\partial\gamma < 0$, $\partial CS^c/\partial\gamma > 0$, and $\partial SW^c/\partial\gamma < 0$, where $\Pi^c$ is industry profit at equilibrium in a CAC system.

Second, from Table 2, the results of Proposition 2 are verified under a TEP system. In particular, we have $\partial t/\partial\gamma < 0$, $\partial q_i^T/\partial\gamma < 0$, and $\partial z_i^T/\partial\gamma < 0$. As the degree of product differentiation decreases or competition increases, profit decreases and consumer surplus increases but social surplus decreases: $\partial\pi_i^T/\partial\gamma < 0$, $\partial\Pi^T/\partial\gamma < 0$, $\partial CS^T/\partial\gamma > 0$, and $\partial SW^T/\partial\gamma < 0$, where $\Pi^T$ is industry profit at equilibrium in a TEP system.

**Table 1. CAC System**

| $\gamma$ | $\lambda_1$ | $\lambda_2$ | $q_1^c$ | $q_2^c$ | $Q^c$ | $z_1^c$ | $z_2^c$ | $\pi_1^c$ | $\pi_2^c$ | $\pi^c$ | $cs^c$ | $sw^c$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 19.80 | 16.50 | 2.600 | 4.250 | 6.850 | 0.423 | 0.647 | 47.35 | 65.50 | 112.9 | 12.41 | 125.3 |
| 0.1 | 19.42 | 16.31 | 2.579 | 4.218 | 6.797 | 0.418 | 0.644 | 46.26 | 64.41 | 110.7 | 13.31 | 124.0 |
| 0.2 | 19.05 | 16.12 | 2.558 | 4.186 | 6.744 | 0.414 | 0.642 | 45.19 | 63.34 | 108.5 | 14.18 | 122.7 |
| 0.3 | 18.68 | 15.93 | 2.538 | 4.155 | 6.693 | 0.409 | 0.639 | 44.15 | 62.30 | 106.4 | 15.01 | 121.5 |
| 0.4 | 18.32 | 15.74 | 2.518 | 4.124 | 6.642 | 0.404 | 0.636 | 43.13 | 61.28 | 104.4 | 15.83 | 120.2 |
| 0.5 | 17.96 | 15.56 | 2.498 | 4.094 | 6.592 | 0.399 | 0.634 | 42.13 | 60.29 | 102.4 | 16.61 | 119.0 |
| 0.6 | 17.61 | 15.38 | 2.478 | 4.064 | 6.542 | 0.395 | 0.631 | 41.16 | 59.32 | 100.5 | 17.37 | 117.8 |
| 0.7 | 17.26 | 15.21 | 2.459 | 4.035 | 6.494 | 0.390 | 0.628 | 40.21 | 58.37 | 98.58 | 18.11 | 116.7 |
| 0.8 | 16.92 | 15.04 | 2.440 | 4.006 | 6.446 | 0.385 | 0.626 | 39.27 | 57.44 | 96.72 | 18.82 | 115.5 |
| 0.9 | 16.58 | 14.87 | 2.421 | 3.978 | 6.399 | 0.380 | 0.623 | 38.36 | 56.54 | 94.90 | 19.51 | 114.4 |
| 1 | 16.25 | 14.70 | 2.403 | 3.950 | 6.352 | 0.376 | 0.620 | 37.47 | 55.65 | 93.12 | 20.18 | 113.3 |

**Table 2. TEP System**

| $\gamma$ | $t$ | $q_1^T$ | $q_2^T$ | $Q^T$ | $z_1^T$ | $z_2^T$ | $\pi_1^T$ | $\pi_2^T$ | $\pi^T$ | $cs^T$ | $sw^T$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 18.00 | 3.500 | 3.500 | 7.000 | 0.286 | 0.857 | 48.25 | 66.25 | 114.5 | 12.25 | 126.8 |
| 0.1 | 17.72 | 3.468 | 3.468 | 6.937 | 0.284 | 0.851 | 47.32 | 64.76 | 112.1 | 13.23 | 125.3 |
| 0.2 | 17.44 | 3.438 | 3.438 | 6.875 | 0.282 | 0.845 | 46.42 | 63.31 | 109.7 | 14.18 | 123.9 |
| 0.3 | 17.16 | 3.407 | 3.407 | 6.814 | 0.280 | 0.840 | 45.54 | 61.90 | 107.4 | 15.09 | 122.5 |
| 0.4 | 16.89 | 3.377 | 3.377 | 6.754 | 0.278 | 0.834 | 44.68 | 60.53 | 105.2 | 15.97 | 121.2 |
| 0.5 | 16.63 | 3.348 | 3.348 | 6.696 | 0.276 | 0.828 | 43.84 | 59.20 | 103.0 | 16.81 | 119.8 |
| 0.6 | 16.37 | 3.319 | 3.319 | 6.638 | 0.274 | 0.822 | 43.02 | 57.90 | 100.9 | 17.62 | 118.5 |
| 0.7 | 16.12 | 3.291 | 3.291 | 6.581 | 0.272 | 0.816 | 42.22 | 56.64 | 98.86 | 18.41 | 117.3 |
| 0.8 | 15.86 | 3.263 | 3.263 | 6.525 | 0.270 | 0.810 | 41.43 | 55.42 | 96.85 | 19.16 | 116.0 |
| 0.9 | 15.62 | 3.235 | 3.235 | 6.471 | 0.268 | 0.805 | 40.67 | 54.22 | 94.89 | 19.89 | 114.8 |
| 1 | 15.38 | 3.208 | 3.208 | 6.417 | 0.266 | 0.799 | 39.92 | 53.06 | 92.98 | 20.59 | 113.6 |

**Table 3. Comparison between CAC System and TEP System**

| $\gamma$ | $\Delta Q$ | $\Delta\pi_1$ | $\Delta\pi_2$ | $\Delta\pi$ | $\Delta cs$ | $\Delta sw$ |
|---|---|---|---|---|---|---|
| 0 | 0.150 | 0.9 | 0.75 | 1.650 | −0.161 | 1.489 |
| 0.1 | 0.140 | 1.06 | 0.35 | 1.417 | −0.075 | 1.342 |
| 0.2 | 0.131 | 1.23 | −0.03 | 1.198 | 0.004 | 1.202 |
| 0.3 | 0.122 | 1.39 | −0.40 | 0.991 | 0.076 | 1.068 |
| 0.4 | 0.113 | 1.55 | −0.75 | 0.797 | 0.141 | 0.939 |
| 0.5 | 0.104 | 1.71 | −1.09 | 0.615 | 0.200 | 0.815 |
| 0.6 | 0.096 | 1.86 | −1.42 | 0.443 | 0.253 | 0.696 |
| 0.7 | 0.088 | 2.01 | −1.73 | 0.282 | 0.300 | 0.582 |
| 0.8 | 0.080 | 2.16 | −2.02 | 0.131 | 0.342 | 0.473 |
| 0.9 | 0.072 | 2.31 | −2.32 | −0.011 | 0.379 | 0.369 |
| 1 | 0.064 | 2.45 | −2.59 | −0.144 | 0.412 | 0.268 |

Finally, we compare the two regulatory policies by considering Table 3, which provides some interesting results as the regulatory regime shifts from a CAC system to a TEP system. First, we have $\partial\Delta Q/\partial\gamma < 0$, which implies that the effects of the regulatory policy change on the increment of total output decreases as competition increases. This is because the trading of permits is limited under severe competition.

Second, we have $\partial\Delta\pi_2/\partial\gamma < 0 < \partial\Delta\pi_1/\partial\gamma$ and $\partial\Delta\Pi/\partial\gamma < 0$. That is, as competition increases, the profit increment of the less (more) efficient firm is increasing (decreasing), but the increment of industry profit is decreasing. This implies that the effect of the profit increment of the more efficient firm on industry profit is larger than that of the less efficient firm. This is so because a more efficient firm can earn greater profit compared to a less efficient firm by participating in permits trading. (Recall that the permit price level is less than half of the total implicit price level (i.e., $t < (\lambda_1 + \lambda_2)/2$)—that is, the average marginal abatement cost of the two firms.)

Finally, we have $\partial\Delta CS/\partial\gamma > 0$ and $\partial\Delta SW/\partial\gamma < 0$: as competition between the two products increases, the increment of total output increases. Thus, the increment of consumer surplus also increases but social welfare decreases. Two remarks are noteworthy. First, the increment of consumer surplus might be negative when the degree of product differentiation is high. In particular, this occurs when $\gamma \le 0.1$ in this numerical example. This implies that the Pareto efficiency of trading permits is restricted under a high degree of product differentiation. Second, the increment of social welfare is always positive irrespective of the degree of product differentiation. In particular, since $\partial\Delta SW/\partial\gamma < 0$ and $\Delta SW > 0$ when $\gamma = 1$, this occurs for all degrees of product differentiation in this numerical example. These two observations imply that as the degree of product differentiation decreases, the net gain in aggregate profits drops faster than the gain in consumer surplus increases.

## 7. Concluding Remarks

In this paper, we consider the case of a duopoly producing a differentiated product and identify the extent of product differentiation as the driving force of our results under the specified environmental policies. In particular, we show that a CAC system gives a greater consumer surplus rather than a TEP system when there is a high degree of product differentiation or less competition between the two firms. We also investigate comparative static effects of the degree of product differentiation on equilibrium output and abatement levels under the two regulatory regimes. These results show that product differentiation can play a significant role in the design and implementation of regulatory policy. Future research may address the role of product differentiation in a number of alternative settings to check the robustness of our results.

## Appendix A

### A.1 Proof of Lemma 1

Substituting $q_i^c$ in (7) and $z_i^c$ in (8) into the binding equation $\overline{E}_i = (\rho - z_i^c)q_i^c$ in (6) yields

$$\lambda_i = \frac{2e_i\rho a(2-\gamma) + 4\rho^3 e_i e_j a - 4e_i e_j \gamma\rho^2 \overline{E}_j - 2(4-\gamma^2)e_i\overline{E}_i - 8e_i e_j \overline{E}_i \rho^2}{4(\rho^2 e_i + 1)(\rho^2 e_j + 1) - \gamma^2}. \quad \text{(A1)}$$

This implies that

$$\lambda_1 - \lambda_2 = \frac{2(e_1 - e_2)(2-\gamma)(a\rho - \overline{E}_1(2+\gamma))}{4 - \gamma^2 + 4\rho^2(e_1 + e_2) + 4\rho^4 e_1 e_2} = \frac{(4-\gamma^2)(e_1 - e_2)(2a\rho/(2+\lambda) - 2\overline{E}_1)}{4 - \gamma^2 + 4\rho^2(e_1 + e_2) + 4\rho^4 e_1 e_2}. \quad \text{(A2)}$$

Since the denominator in (A2) is positive, the sign of $\lambda_1 - \lambda_2$ is determined by the sign of the last term in the numerator, $2a\rho/(2+\gamma) - 2\overline{E}_1$, when $e_1 > e_2$. Since $q_1^c + q_2^c = (2a - \rho(\lambda_1 + \lambda_2))/(2+\gamma)$ in (9) and $\lambda_i$ is positive, we know that $\rho(q_1^c + q_2^c) < 2a\rho/(2+\gamma)$. Then, since $z_i^c$ is positive, we have $\rho q_1^c > \overline{E}_1$ from (6). Thus, $\rho(q_1^c + q_2^c) > 2\overline{E}_1$. Therefore, combining these two inequalities leads to $2\overline{E}_1 < \rho(q_1^c + q_2^c) < 2a\rho/(2+\gamma)$.

### A.2 Proof of Lemma 2

First we know that the total emission quota, $\overline{E}$, under a TEP system is the same as that under a CAC system, in which $\overline{E} = \overline{E}_1 + \overline{E}_2 = (\rho - z_1^c)q_1^c + (\rho - z_2^c)q_2^c$ at equilibrium. Substituting the equilibrium characterized in (7) and (8) under a CAC system into $\overline{E}$ and rewriting (17) yields

$$t = \frac{e_2(2e_1\rho^2 + (2+\gamma))\lambda_1 + e_1(2e_2\rho^2 + (2+\gamma))\lambda_2}{(2+\gamma)(e_1 + e_2) + 4\rho^2 e_1 e_2}. \quad \text{(A3)}$$

Letting $B_i = 2e_1 e_2 \rho^2 + e_i(2+\gamma)$, we can rewrite (A3) as

$$t = \frac{B_2}{B_1 + B_2}\lambda_1 + \frac{B_1}{B_1 + B_2}\lambda_2 . \tag{A4}$$

This indicates that the emission permit price *t* is a linear combination of implicit prices, $\lambda_1$ and $\lambda_2$. From Lemma 1, where $\lambda_1 > \lambda_2$ when $e_1 > e_2$, we have $0 < \lambda_2 < t < \lambda_1$. Notice also that if $e_1 > e_2$, then $B_1 > B_2$. Thus, $B_1/(B_1 + B_2) > 1/2$. This implies that the weight for $\lambda_1$ is smaller than that for $\lambda_2$, i.e., $t < (\lambda_1 + \lambda_2)/2$.

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