A Simple Model of Reliability, Warranties, and Price-Capping

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Abstract

The paper presents a model of reliability which captures the process by which reliability is actually determined more accurately than the conventional analysis. In contrast to that conventional analysis, which is based on the characteristics approach, the model of this paper defines reliability as the objective probability of product failure, not as a characteristic of individual goods. Reliability, thus defined, is treated as a choice variable of the firm. The resulting model is applied to a monopolist subject to a price cap. The monopolist can vary reliability and the terms of a warranty or compensation deal in response to price-capping. The monopoly outcome, price-capped monopoly outcome, and Pareto-efficient outcome are compared. The model provides a theoretical explanation of some empirical results in the literature on electricity regulation.

Key words: reliability; warranties; price-capping

JEL classification: L15; L51; M21

1. Introduction

Monopolies are often subject to some form of price-capping. In the UK, for example, the utilities (telecoms, electricity, gas, and water) all have regulators who frequently exercise their powers to cap prices. One would expect that the rational, profit-maximising monopolist would respond to a binding price cap by changing other variables, in an attempt to shift the demand and cost curves in his favour. Obvious candidates for this role are (a) reliability of the product or service and (b) the contractual terms on which the product or service is offered to the market; for example, the terms of any warranty or compensation deal which the monopolist may provide along with its output. Throughout the paper the generic term "output" will be used instead of "product" or "service." There is some relevant empirical evidence

on this matter. For example Fraser (1994a, b) considers the supply of electricity by a private monopoly, showing that a pure price cap induces the firm to reduce reliability of supply.

In order to analyse this problem theoretically it is necessary to develop a model which allows the firm to determine reliability and the terms of any warranty or compensation deal as part of its overall profit-maximising decision. Unfortunately, the existing theoretical literature does not provide much of a guide on these matters, partly because the conventional analysis of output quality is based on the characteristics approach to modelling production. This paper draws a sharp distinction between quality and reliability. It defines the latter as the objective probability (relative frequency) of output failure, a probability which the firm can choose, for example via the quality control process, the choice of technique, or the design of output. The paper argues that this approach comes closer to describing the process by which reliability is actually determined than does the conventional output quality approach, which is more relevant to aspects of production other than reliability. For example, the conventional analysis explains well why some computers have fast processors or large hard drives, but not why they break down so often.

The paper develops this analysis of reliability and incorporates it into a simple model of monopoly, which is then applied to the price-capping problem. Section 2 develops the model of reliability. In Section 3 the demand side is developed, assuming risk averse consumers, with a reservation price varying across the market. Section 4 focuses on the supply side, incorporating reliability costs and warranty costs into the monopolist's profit function. In Section 5 a Pareto-efficient allocation of resources is characterised. Sections 6 and 7 characterise monopoly equilibrium without and with a price cap, respectively. Comparing these two types of equilibrium with each other and with the Pareto-efficient allocation allows an analysis of the effects of price-capping. The mathematical details are relegated to an Appendix. Section 8 concludes.

2. Modelling Reliability

In the model presented here "reliability" will be defined as the probability of an event disliked by consumers not occurring. Examples of such an event are:

A: A consumer durable (e.g., computers, cars, washing machines, or televisions) breaking down within some given time period.

B: An intermediate good such as a silicon chip (or other component) failing to function correctly (here the consumer is another firm).

C: A train or plane not arriving within some predetermined time period of its scheduled arrival time.

D: Electricity, gas, water supplies, or telephone services being interrupted for more than some predetermined period. Note that commercial contracts for these services may allow some interruption of service without penalty (e.g., the commercial supply of gas). Matsukawa and Fujii (1994) study Japanese electricity

consumers and show, among other things, that they face a trade-off between price and reliability of electricity supply.

The probability referred to here is an objective relative frequency. For example, if a computer manufacturer produces 100,000 computers each year and 93,000 do not break down within a given time period (say one year), the reliability of these computers is 0.93. Thus the computer firm knows for sure that 7,000 of its computers will break down within the year, but it neither knows nor cares which 7,000 they will be. Now suppose the firm offers a one-year warranty with its computers, promising compensation in the event of a breakdown. In this model the firm faces no uncertainty concerning its profits: it knows its revenue and production costs, it knows that there will be 7,000 claims under the warranty (though not which customers will make them), and it knows how much it will have to pay out per claim (that can either be treated as endogenous or imposed by a regulator). There is therefore no uncertainty about its profits.

It will be assumed that consumers (who, in the case of intermediate goods mentioned in point B above, will be other firms) have no knowledge about individual products or services but do know the reliability of each firm's output (in the sense defined here). Consumers read *Which?* magazine or *Consumer Reports* or obtain information on reliability from other sources. For example, in the UK, the Strategic Rail Authority publishes information on the average punctuality of the different rail operators.

Supplying firms will be assumed to have the same information. This is a plausible assumption because it is usually impossible or extremely costly for firms to obtain information on each example of its output before it is sold. Firms will be assumed to vary reliability (as defined here), for example via product design, choice of technique, and quality control procedures. It will be assumed that higher reliability entails higher production costs. Thus, the computer manufacturer will be able to increase or decrease the number of breakdowns in a given time period without knowing (or caring) which computers will break down and which will not. It will therefore be assumed to know the reliability of its output (as defined here), without knowing which examples of its output will break down.

In this model the firm faces no uncertainty, though this is not true of consumers, who are assumed to be risk averse. This assumption is readily justified, for example, in the consumer durables market, where each consumer typically owns one example of the good and is thus extremely concerned at the prospect of its breaking down. The firm, by contrast, supplies many examples of the good and may well find it profitable to operate a risk-pooling warranty scheme. Under these assumptions there arises a demand, on the part of consumers, for insurance. This might, as mentioned above, be provided in the form of a product warranty offered by the firm or an insurance policy provided jointly with the product. In the case of intermediate goods, "warranties" may be thought of as compensation clauses built into standard supply contracts. A similar interpretation applies to services such as electricity and gas. In the case of transport services, it is clearly possible for suppliers to offer compensation to dissatisfied passengers. Throughout the paper attention will be confined to voluntarily offered

warranties or compensation, though the model is readily modified to include legally compelled compensation. It could also be modified to cover more than one undesired event (e.g., different degrees of product breakdown) or to cover product hazard and safety issues. Warranties, whether voluntary or legally compelled, have an important bearing on decisions affecting reliability because the higher the reliability of a firm's marketed output, the lower the warranty costs experienced by the firm *ceteris paribus*. In the model developed in this paper, warranties play the role of allocating risk and providing an incentive to supply reliable output, in contrast to much of the existing literature on warranties where they have a signalling role.

As noted in Section 1, the model developed in this paper differs sharply from that presented in the literature on product quality. That literature deals either with search goods or with experience goods. In the former case both supplier and consumer know all relevant characteristics of each good or service before sale takes place (e.g., the computer has a 3.0GHz processor and an 80Gb hard drive, or the train has a good restaurant). See for example Mussa and Rosen (1978) and Matthews and Moore (1987) for models of this type.

In the case of experience goods, there is an asymmetry of information. Nature dictates all relevant characteristics of each good or service to the supplier before sale, but these are unknown to the consumer at that stage (e.g., the computer's hard drive will fail in the first year, or the train will be 2 hours late). The supplier's problem is thus one of signalling. Perhaps by means of advertising, or offering a warranty or compensation deal, the supplier of high quality output seeks to signal his high quality to consumers in a credible way. See for example Grossman (1981), Milgrom and Roberts (1982, 1986), Kreps and Wilson (1982), Klein and Leffler (1981), Shapiro (1983), and McClure and Spector (1991) for models of this type.

Neither of these approaches is of much use in analysing reliability. Search good models assume too much information on both sides of the market, while experience good models assume too much information on the supply side and not enough on the demand side. Moreover, the latter kind of model is based on an exogenously given quality level, while reliability (as defined in this paper) will be determined endogenously by the supplier's decisions, as discussed above.

A standard problem, often assumed away in the literature, is that of moral hazard on the part of consumers. If consumers can themselves influence the probability or size of a claim under the warranty, for example by failing to take proper care of the good during consumption, then the economic role of warranties may be reduced. See for example McKean (1970), Oi (1973), Priest (1981), and Goering (1997), who discusses the problem of moral hazard facing a durable goods monopolist. For simplicity moral hazard will be assumed away in this paper. It should be noted that the model presented here focuses on reliability and warranties, deliberately suppressing some other aspects of the markets discussed above. It is essentially a static model, and is not intended to deal with the issue of dynamic consistency in durable goods markets. Moreover, it is a model of symmetric information. In such a model nothing can be gained by admitting the possibility of repeat purchasing since neither side of the market can learn anything useful about the other.

3. The Demand Side

In our model, consumers' preferences have three distinct aspects:

1. The consumer's preference for the good in its working state. This varies across consumers and is exogenous.
2. The consumer's degree of risk aversion. For convenience this is assumed constant across consumers and is exogenous.
3. The probability of the good not breaking down within some given time period (i.e., the reliability of the good). Subjective and objective probabilities are, by definition, identical in this model. It is an essential feature of the model that this probability is endogenous (determined by firms' decisions) and the same for all consumers.

The details of consumers' utility functions are developed below.

The demand side of the market will be assumed to consist of z consumers, each consuming a single unit of the output. Each consumer has a different reservation price, and hence the market demand curve is downward sloping. For simplicity we take z to be a strictly positive real variable. Each consumer has a money budget M available and pays a price p for the output. As discussed in Section 2, two states of the world are assumed: either the undesired event occurs or it does not. In the latter case, the z th consumer receives a stream of services which she values at $f(z)$ (perhaps generated by a durable good). Note that $z > 0$ and $f'(z) < 0$. In the former case the consumer values the stream of services at zero, but the firm makes a voluntary warranty (or compensation) payment of β to her. Costs of writing and enforcing the warranty (or compensation) contract are ignored. Thus the z th consumer receives income stream:

$$x = M - p + f(z) \tag{1}$$

if the undesired event does not occur, and

$$y = M - p + \beta \tag{2}$$

if it does.

The reliability of a product will be defined as in Section 2, as the objective probability R of the undesired event not occurring. Consumers are assumed to be risk-averse maximisers of expected utility. As discussed in Section 2, it will be assumed that consumers are fully informed about reliability, so that the subjective probability of the undesired event not occurring is equal to the objective probability. Of course R is determined endogenously by the management decisions of the monopolist.

The z th consumer maximises expected utility:

$$V = RU\big(M - p + f(z)\big) + (1 - R)U(M - p + \beta). \tag{3}$$

Clearly $U'(\cdot) > 0$, and, to ensure risk aversion, it is assumed that $U''(\cdot) < 0$ (i.e., the function $U(\cdot)$ is assumed strictly concave).

Note that the z th consumer is indifferent between consuming and not consuming when:

$$V = RU\big(M - p + f(z)\big) + (1 - R)U(M - p + \beta) = U(M) \tag{4}$$

since $U(M)$ is the expected utility she would get by not consuming the output (she will be referred to as the "marginal consumer"). Equation (4) generates, for given values of R and β, a relationship between p and z, namely the market demand curve. Each consumer has a different reservation price, and thus the market demand curve slopes downwards (see Figure 1). Note that R and β are determined by the decisions of the monopolist, so that consumers can be thought of as consuming a bundle consisting of a stream of services (perhaps provided by a durable good), its reliability, and the warranty deal. They are not able to unbundle these three parts. If the monopolist raises R or β, the demand curve will shift upwards, except that, when a full money back warranty is offered ($\beta = p$), the marginal consumer will be indifferent as to whether the undesired event occurs or not (since $x = y$ when $\beta = p$). In this case, as R changes, the demand curve will rotate about the equilibrium, which will itself be immune to variations in R. Note also that the $f(z)$ curve must be steeper than the demand curve (see Figure 1) because it is the relationship between p and z which would hold if β were continually kept equal to p (this is clear from equation (4)). Of course the demand curve is defined *ceteris paribus* (i.e., holding everything constant except p and z).

Figure 1. The Distribution of Tastes ($f(z)$) and the Market Demand Curve ($D(z,R,\beta)$).
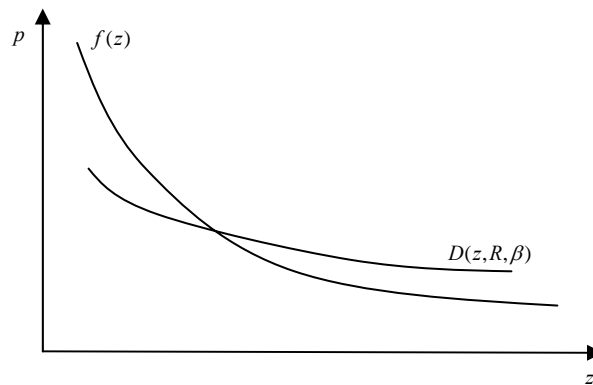
4. The Supply Side

Firms' costs will depend on the reliability of their output for a number of different reasons:

1. Reliability can be designed into the output. Higher reliability designs will, in general, be more costly to produce than lower reliability ones.
2. Techniques of production can be adopted which generate higher reliability. Techniques generating higher reliability output will, in general, be more costly to operate than those generating lower reliability output.
3. The stringency of quality control can be varied. Stricter quality control will, in general, raise reliability, but will also raise scrap or rework costs.
4. Higher reliability will reduce the number of claims under the warranty and hence, for a given warranty payment, reduce warranty costs.

The model developed here formalises these costs by assuming that production costs are increasing in the reliability of output and by incorporating warranty costs into the firm's profit-maximising decision. Average and marginal production costs, at a given reliability level, will be assumed constant. Note that z is the monopolist's output.

Adopting the assumptions set out above, a suitable production cost function is:

$$zC(R),\tag{5}$$

where $C'(R) > 0$ and $C''(R) > 0$ for $0 < R < 1$. The number of times that the undesired event occurs is clearly $z(1-R)$, and thus warranty or compensation costs are given by:

$$\beta z(1-R).\tag{6}$$

Thus the monopolist maximises the profit function:

$$\Phi = pz - zC(R) - \beta z(1-R).\tag{7}$$

5. Pareto-Efficiency

We now characterise a Pareto-efficient allocation of resources. Since there are no non-convexities, externalities, or public goods in the model, a Pareto-efficient allocation of resources would be brought about by the operation of a perfectly competitive market. A Pareto-efficient allocation is defined as a 4-tuple (p, z, R, β) that maximises each consumer's expected utility subject to the constraint that profits are non-negative. This problem is easily solved by taking a multiplier (λ) for the profit constraint and forming the Lagrangian:

$$L = RU(M - p + f(z)) + (1-R)U(M - p + \beta) + \lambda(pz - zC(R) - \beta z(1-R)).\tag{8}$$

By considering the first-order conditions of this problem, two important results can be obtained. The proofs are relegated to the Appendix. Note that an asterisk denotes the efficient level of a variable.

First, the profit-constraint is binding; that is, Pareto-efficiency requires zero (supernormal) profits and consumers extract all the surplus (see Appendix,

Proposition 1). In the absence of monopoly this would be achieved by free entry. Note that price discrimination by the monopolist is assumed away, so one would not expect monopoly equilibrium to be Pareto-efficient. Second, Pareto-efficiency requires that the marginal consumer is fully insured (Appendix, Proposition 2). Thus a full money back warranty must be offered (i.e., $\beta^* = p^*$ so that $x = y$ and the marginal consumer is indifferent as to whether the undesired event occurs or not).

6. Monopoly Equilibrium

In monopoly equilibrium there is a single supplier maximising her profits subject to the voluntary participation constraint. This is the constraint that each consumer obtains at least as much expected utility from purchasing the output as from not doing so. Mathematically it is simply:

$$RU(M - p + f(z)) + (1 - R)U(M - p + \beta) \geq U(M) . \qquad (9)$$

In equilibrium z is determined at a level which makes this constraint bind (i.e., the z th consumer is the marginal consumer, who is just on the point of leaving the market and z is the monopolist's total output). A monopoly equilibrium is easily characterised by taking a Lagrange multiplier (μ) for the constraint (9) (noting equation (7), which specifies the monopolist's profits) and forming the Lagrangian:

$$M = pz - zC(R) - \beta z(1 - R) + \mu(RU(x) + (1 - R)U(y) - U(M)) . \qquad (10)$$

From the first-order conditions for this problem the following results can be obtained (proofs in Appendix). First, the voluntary participation constraint is binding (Appendix, Proposition 3). This allows the monopoly output (z^m) to be determined. Note that the superscript m denotes the value of a variable in monopoly equilibrium without price-capping and the superscript c denotes the value of a variable in monopoly equilibrium with price-capping. Second, in monopoly equilibrium without price-capping, risk is efficiently allocated (i.e., the marginal consumer is fully insured, so that $p^m = \beta^m$; see Appendix, Proposition 4). The monopoly equilibrium is illustrated in Figure 2.

Manipulating the first-order conditions allows a comparison between the monopoly levels of the relevant variables (without a price cap) with their efficient levels. In particular:

$$R^m > R^* ; \ \beta^m > \beta^* ; \ p^m > p^* ; \ z^m < z^* . \qquad (11)$$

See Appendix, Proposition 5 and Corollaries 2, 3, and 4. The monopolist raises reliability above its efficient level but lowers output and raises price in the usual way. As remarked above, it allocates risk efficiently.

Figure 2. M **Is the Uncapped Monopoly Equilibrium,** C **Is the Capped Monopoly Equilibrium,**
and E **Is the Pareto-Efficient Outcome.**
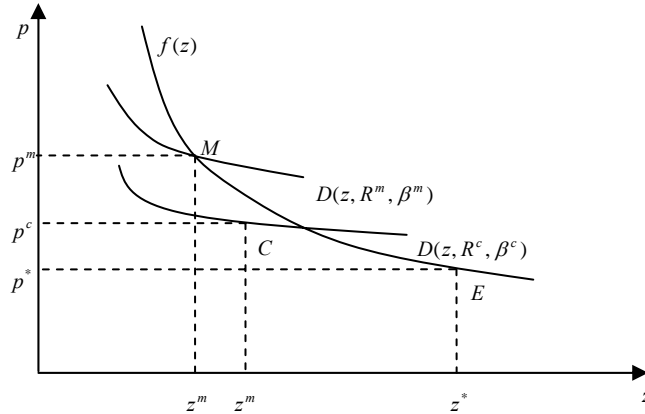
7. Monopoly Equilibrium with Price-Capping

The monopoly model of Section 5 illustrates the standard sources of inefficiency under monopoly, namely a restricted output and raised price (relative to the efficient levels). In addition to this, inefficiency arises because the monopolist supplies a reliability level above the efficient level. However, the monopolist does allocate risk efficiently by offering a full money back warranty. In this section, we consider the consequences of a binding price cap. Suppose a binding price cap (p^c) is imposed on the monopolist such that $p^m > p^c \geq p^*$. A reduction in reliability will clearly lower costs and may therefore increase the firm's profits. With a money back warranty in place, the marginal consumer is indifferent as to whether the undesired event occurs or not, and consequently the market equilibrium will be immune to variations in reliability. In fact, in the model set out above, the effect of price-capping is rather more complex than this because it induces under-insurance and an inefficient allocation of risk. Because of the under-insurance, the market demand curve now shifts in response to changes in R (rather than simply rotating about the equilibrium). In particular, lowering R shifts the demand curve downwards (see Figure 2).

The analysis of price-capping is best approached by modifying the Lagrangian of Section 5 (equation (10)), by adding a constraint on the price:

$$p \leq p^c. \tag{12}$$

We take a multiplier ν for this constraint, modifying equation (10) as follows:

$$M = pz - zC(R) - \beta z(1-R) + \mu(RU(x) + (1-R)U(y) - U(M)) + \nu(p^c - p). \tag{13}$$

The multiplier v represents the monopolist's marginal valuation of the price cap, i.e., the amount by which she could increase her profits if the price cap were relaxed by one marginal unit (or the maximum she would be willing to pay as a bribe in order to get a marginal, one-unit increase in the price cap). We are interested, in this paper, in effective price-capping and will therefore assume that, under price-capping, $v > 0$. Note that equation (13) includes the uncapped case (i.e., when $v = 0$). In the Appendix, the first-order conditions of the Lagrangian (equation (13)) are derived, thus incorporating the price-capped and uncapped cases into the same mathematical problem.

Using these first-order conditions the monopoly equilibrium with price-capping ($v > 0$) can be characterised. Comparing this equilibrium with the uncapped case ($v = 0$), the effects of price-capping can be deduced. In particular, monopoly output with price-capping is below the efficient level (Appendix, Proposition 6). Moreover, price-capping induces the monopolist to provide a less-than-money-back warranty (Appendix, Proposition 4 and Corollary 1). Thus, the marginal consumer is not fully insured and risk is not efficiently allocated. The market demand curve now shifts in response to changes in reliability. Price-capping also induces the monopolist to reduce reliability (Appendix, Proposition 7), which, as noted above, was inefficiently high initially. The cost savings associated with the reliability reduction and the under-insurance are partly passed to consumers via the lower price and partly taken by the monopolist as (supernormal) profits.

8. Conclusions

This paper has introduced a new definition of reliability based on the objective probability of output failing in the hands of consumers. This definition has been incorporated into a simple model of monopoly in which the firm can vary output reliability and has an incentive voluntarily to offer a warranty (or compensation deal) to the market. The model suggests that, even though the imposition of an effective price cap on the monopolist has the standard effect of increasing quantity and decreasing price, it will also induce the monopolist to reduce reliability. This is consistent, for example, with the findings of Fraser (1994a, b), who shows that a pure price cap imposed on a private monopoly supplier of electricity induced the firm to reduce the reliability of supply. The model also implies that an uncapped monopolist will offer a full money back warranty, thus allocating risk efficiently. An effective price cap induces the monopolist to worsen the warranty (or compensation) terms, generating under-insurance and an inefficient allocation of risk. The cost savings arising from reliability reduction and under-insurance are partly passed on to consumers via the lower price and partly taken by the monopolist as (supernormal) profits.

Appendix

This Appendix contains the proofs of the results discussed in the main text. Part A characterises a Pareto-efficient allocation of resources. Part B characterises

monopoly equilibrium with and without a price cap. The approach is to derive and utilise the first-order conditions of the Lagrangians given in the main text.

A. Pareto-Efficient Allocation

Consider first the Lagrangian of equation (8). Its first-order conditions characterise a Pareto-efficient allocation:

$$L = RU(M - p + f(z)) + (1 - R)U(M - p + \beta) + \lambda(pz - zC(R) - \beta z(1 - R)). \quad \text{(A1)}$$

It is easy to prove the following proposition.

Proposition 1. Pareto-efficiency requires that the profit constraint is binding (i.e., $\Phi = 0$).

Proof. Differentiating equation (A1) with respect to p and using equations (1) and (2) yields one of the first-order conditions for an interior maximum:

$$\begin{aligned}
&L_p = -RU'(x) - (1 - R)U'(y) + \lambda z = 0 \\
&\Rightarrow \lambda z = RU'(x) + (1 - R)U'(y) > 0.
\end{aligned} \quad \text{(A2)}$$

But $z > 0$, hence $\lambda > 0$, and it follows by complementary slackness that the profit constraint is binding.

We now establish that Pareto-efficiency requires that a full money back warranty is offered. First it is necessary to establish three useful lemmas.

Lemma 1. Pareto-efficiency requires that $\beta = f(z)$.

Proof. Differentiating equation (A1) with respect to β and using equations (1) and (2) yields another first-order condition for an interior solution:

$$\begin{aligned}
&L_\beta = (1 - R)U'(y) - \lambda z(1 - R) = 0 \\
&\Rightarrow (1 - R)U'(y) = (1 - R)(RU'(x) + (1 - R)U'(y)) \\
&\Rightarrow U'(y) = RU'(x) + (1 - R)U'(y) \\
&\Rightarrow RU'(y) = RU'(x) \\
&\Rightarrow x = y,
\end{aligned}$$

using equation (A2), since $R \neq 1$ and $R \neq 0$ because we are seeking an interior solution, and $U'(.)$ is invertible because $U''(\cdot) < 0$. Hence from equations (1) and (2) we have $\beta = f(z)$.

Lemma 2. Pareto-efficiency requires that $\beta = C'(R)$.

Proof. Differentiating equation (A1) with respect to R and using equations (1) and (2) yields another first-order condition for an interior solution:

$$L_R = U(x) - U(y) + \lambda(-zC'(R) + \beta z) = 0. \quad \text{(A3)}$$

But $x = y$ (from Lemma 1), $\lambda > 0$ (from Proposition 1), and $z > 0$ (by definition), hence equation (A3) implies that $\beta = C'(R)$ as required.

Lemma 3. Pareto-efficiency requires that $p = f(z)$.

Proof. We have $x = y$ from Lemma 2. So equation (4) yields:

$$RU(x) + (1-R)U(x) = U(M)$$
$$\Rightarrow U(x) = U(M)$$
$$\Rightarrow x = M \; ,$$

using the fact that $U(\cdot)$ is invertible because $U'(\cdot) > 0$. Hence, from equation (1) $p = f(z)$ as required.

It is now straightforward to establish Proposition 2.

Proposition 2. Pareto-efficiency requires that a full money back warranty is offered (i.e., $p = \beta$).

Proof. Combining Lemmas 1 and 3, we have $p = \beta$ as required.

We now establish the following useful lemma.

Lemma 4. Pareto-efficiency requires that $C'(R)R = C(R)$.

Proof. From Proposition 1, Pareto-efficiency requires that profits are zero. Thus, using Proposition 2 and equation (7) we obtain:

$$pz - zC(R) - pz + pzR = 0$$
$$\Rightarrow pR = C(R) \; .$$

Combining Proposition 2 and Lemma 2, we have $p = \beta = C'(R)$. Thus, $C'(R)R = C(R)$ as required.

B. Monopoly Equilibrium

In this section we characterise monopoly equilibrium with and without price-capping by deriving the first-order conditions of the Lagrangian specified in Section 6 of the main text as equation (13):

$$M = pz - zC(R) - \beta z(1-R) + \mu(RU(x) + (1-R)U(y) - U(M)) + \nu(p^c - p). \quad \text{(A4)}$$

Note $\nu > 0$ corresponds to the price-capped case and $\nu = 0$ to the uncapped case.

Differentiating (A4) yields first-order conditions for an interior solution. Differentiating with respect to p yields:

$$z + \mu\left(-RU'(x) - (1-R)U'(y)\right) - \nu = 0 \; . \tag{A5}$$

Differentiating with respect to β yields:

$$-z(1-R)+\mu(1-R)U'(y)=0.$$

(A6)

Differentiating with respect to R yields:

$$-zC'(R)+\beta z+\mu[U(x)-U(y)]=0.$$

(A7)

Differentiating with respect to z yields:

$$p-C(R)-\beta(1-R)+\mu RU'(x)f'(z)=0.$$

(A8)

It is now straightforward to establish the following results.

Proposition 3. In monopoly equilibrium (price-capped and uncapped) the voluntary participation constraint binds. That is:

$$U(M)=RU(x)+(1-R)U(y).$$

(A9)

Proof. Equation (A6) implies that $\mu \neq 0$ since we are seeking an interior solution (so that $z>0$ and $1>R>0$). Hence, by complementary slackness, the voluntary participation constraint must bind.

Proposition 4. The uncapped monopolist provides the marginal consumer with full insurance, while the price-capped monopolist provides under-insurance.

Proof. Equations (A5) and (A6) together imply that $R(U'(y)-U'(x))=v/\mu$. Thus, for the unregulated case ($v=0$) we must have $x=y$. (Note that $U'(\cdot)$ is invertible because $U''(\cdot)<0$.) Hence, in the uncapped case, the marginal consumer is fully insured. For the price-capped case we have $v>0$ and hence, by a similar argument, $y>x$ and the marginal consumer is less than fully insured.

Corollary 1. For the uncapped case we have:

$$f(z)=p=\beta,$$

(A10)

and for the price-capped case:

$$f(z)>p>\beta.$$

(A11)

Proof. These results follow from (A9) and Proposition 4.

We now prove the inequalities in (11) of the main text which express comparisons between monopoly levels (with and without a price cap) and efficient levels of the relevant variables. We start by considering reliability R.

Proposition 5. Reliability in uncapped monopoly equilibrium is above the Pareto-efficient level.

Proof. Equations (A6) and (A8) together imply that:

$$p + \frac{zRU'(x)}{U'(y)} f'(z) = C(R) + \beta(1-R) . \tag{A12}$$

But in the uncapped case $x = y$, hence (A12) implies:

$$p + zRf'(z) = C(R) + \beta(1-R) . \tag{A13}$$

Now note that, given $x = y$, (A7) implies that:

$$\beta = C'(R) . \tag{A14}$$

Combing (A13), (A14), and Corollary 1 yields:

$$C(R) - RC'(R) = zRf'(z) . \tag{A15}$$

Now define the function $F(R)$ as follows:

$$F(R) = C(R) - RC'(R) . \tag{A16}$$

Differentiating F (noting that $C''(.) > 0$) gives:

$$F'(R) = -RC''(R) < 0 . \tag{A17}$$

Using the function F we can now compare the monopoly level of R with the efficient level. From Lemma 4:

$$F(R^*) = 0 , \tag{A18}$$

while (A15) gives:

$$F(R^m) < 0 , \tag{A19}$$

since $f'(z) < 0$. Hence (A17) implies that $R^m > R^*$ as required.

Corollary 2. $\beta^m > \beta^*$.

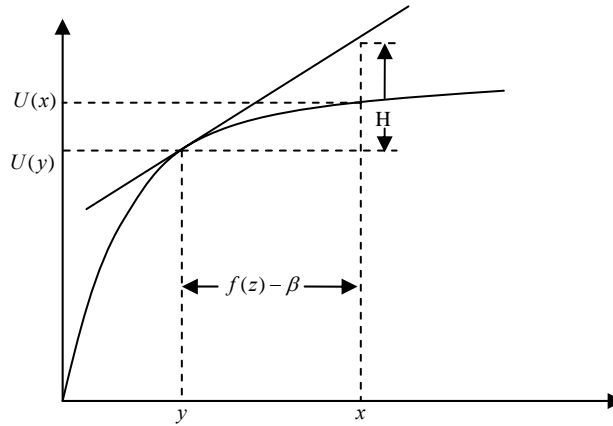***Proof.*** Corollary 2 follows from Lemma 2, (A14), and Proposition 5 and noting that $C'(R) > 0$.

Corollary 3. $p^m > p^*$.

Proof. Corollary 3 follows from Proposition 2, (A10), and Corollary 2.

Corollary 4. $z^m < z^*$.

Proof. Corollary 4 follows from Lemma 3, (A11), and Corollary 3.

Figure 3. The Function $U(\cdot)$.

We now turn to the price-capped monopolist. It is useful to establish the following lemma.

Lemma 5. Under price-capped monopoly $C'(R) < f(z)$.

Proof. Using (A6) to eliminate μ from (A7) yields:

$$C'(R) = \beta + \frac{U(x) - U(y)}{U'(y)}. \tag{A20}$$

The proof relies on the strict concavity of the utility function $U(\cdot)$; see Figure 3. Strict concavity implies that:

$$
\begin{aligned}
&U(x) - U(y) < H < U'(y)(x - y) = U'(y)(f(z) - \beta) \\
&\Rightarrow \frac{U(x) - U(y)}{U'(y)} < f(z) - \beta.
\end{aligned} \tag{A21}
$$

Combining (A20) and (A21) yields the result required.

We can now establish the following proposition.

Proposition 6. The price-capped monopoly output is below the efficient level.

Proof. Combining Lemma 3 with (A11) yields:

$$f(z^c) > p^c \geq p^* = f(z^*) . \qquad (A22)$$

Hence, noting that $f'(.) < 0$, it follows that $z^c < z^*$ as required.

Finally we establish Proposition 7.

Proposition 7. Reliability is lower under price-capped monopoly than under uncapped monopoly.

Proof. Combining Lemma 5, (A10), and (A14) yields:

$$C'(R^c) < f(z^c) < f(z^m) = C'(R^m) , \qquad (A23)$$

noting that $f'(\cdot) < 0$ and $C'(\cdot) > 0$. Hence $R^c < R^m$ as required.

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