Limit-Pricing and Learning-By-Doing:
A Dynamic Game with Incomplete Information

Ke Yang[*]

Barney School of Business, University of Hartford, U.S.A.

Abstract

We study a firm's pricing/output strategy under threat of entry in a two-period game with asymmetric information, where the firm can reduce future cost through learning-by-doing. In contrast with previous literature, we show that a firm's incentive to reduce cost through higher production may not align with its incentive to signal its cost type. As a consequence, in equilibrium, the incumbent firm might distort its price upward instead of downward.

Key words: limit-pricing; learning-by-doing; dynamic game

JEL classification: L11; L12; L13

1. Introduction

"Limit-pricing" was first introduced in Bain (1949) to explain "a prolonged tendency to hold price well below the level which would maximize the difference between aggregate revenue … and the aggregate cost …." It is suggested that the market incumbent may be able to use price alone to discourage further entry because price may indicate the profitability of the market. As such, limit-pricing has constituted a major theme in industrial literature for the last forty years. Many studies, employing behavior models to explore the decision problem of the firm, take as given the limit-pricing assumption, i.e., that a lower pre-entry price will deter or limit entry.

However, the argument underlying the limit-pricing concept is controversial because the commitment value of pricing is doubtful. Price is flexible in the long run, so potential entrants should not be discouraged by a low pre-entry price. Once entry occurs, price will adjust to represent the true profitability of the market, and potential entrants should be able to see this at the time they make their entry decision. It is not clear, by this logic, why limit-pricing emerges at all. The earliest treatment of this problem is the Bain-Sylos-Modigliani model (Bain, 1993; Labini,

1962; Modigliani, 1958), where the potential entrant is assumed to believe that the incumbent firm would maintain the same output level after entry as its pre-entry output. Then the incumbent firm naturally gets a Stackelberg leadership role in the post-entry market. This gives the incumbent firm the incentive to lower the current market price. Nonetheless, the assumption about the potential entrant's belief is dubious for the same reason.

There have been major advances in developing the foundations for limit-pricing theory. These advances have attributed the observed lower pre-entry price either to the commitment value of capital investment or information asymmetry. The commitment value of capital investment explanation suggests that a threat that is costly to carry out can be made credible by entering into an advance commitment. Dixit (1979, 1980) and Spence (1977) re-interpret limit-pricing as an outcome of capacity competition. They suggest that by accumulating a large capacity in the pre-entry period, the incumbent can gain a competition advantage over the potential entrant in the post-entry market, thereby limiting the rate of entering or deterring entry completely. The capacity accumulation over the short-run optimal level leads to a lower pre-entry price. Many business practices can be used as capital investment to deter entry, such as developing a faithful clientele or setting up a network of exclusive franchises.

The learning-by-doing (LBD) effect can be used as another form of capital investment to erect barriers to entry. It has been widely verified that in many industries, the experiences acquired by the incumbent during pre-entry periods reduce their current costs and thus may be considered as a form of capital. The cost reduction through LBD gives the incumbent firm a competitive advantage over the prospective entrant and thereby discourages entry. For this reason, the incumbent, to gain a bigger market share in the post-entry market, may strategically increase their production even further in the pre-entry period, leading to a lower pre-entry price.

Attributing lower price to commitment value of capital investment diverts from the original spirit of "limit-pricing" in Bain (1993), which states that price alone indicates the profitability of the market. Can a lower price alone be used as a tool to discourage entry, even though it is flexible and has no commitment value? As shown in Milgrom and Roberts (1982), under information asymmetry regarding the profitability of the market, a lower price tends to discourage entry by signaling lower costs. So, this explanation extends the original idea of "limit-pricing" in Bain (1993). In Milgrom and Roberts (1982), the lower price is a consequence of the incentive to signal due to information asymmetry, and this has become the current meaning of "limit-pricing."

While both capital accumulation (e.g., LBD) and signaling cost can lead to a lower pre-entry price, it is not clear how the pricing strategy changes for the incumbent if one considers simultaneously (a) that price may be used to signal profitability of the market when information is asymmetric and (b) the LBD effect can be exploited by the incumbent firm to erect entry barriers. Can we expect an even lower pre-entry price compared to the case where only one of the strategies is available to the incumbent firm?

Lee (1975) and Smiley and Ravid (1983) study a dynamic model combining both LBD and limit-pricing to answer the above question. However, they both take as given the assumption that lower price does limit entry; they do not explore how the underlying incentives may interact with the strategic use of LBD. The model in this paper differs from Lee (1975) and Smiley and Ravid (1983) in that instead of assuming that lower price limits entry, we assume only an information-asymmetric environment in which price could be used by the incumbent to signal its cost type. Our objective is to investigate, in this context, how a rational economic agent who can learn from experience responds to the threat of entry. In contrast with the conventional limit-pricing model, we show that under certain circumstances the incumbent might price higher than it would under complete information.

The paper is organized as follows. In next section, we review a simple game under complete information to illustrate how the incumbent can use LBD to limit or deter entry. In Section 3, we study a two-period game with LBD but under asymmetric information. Concluding remarks are given in Section 4.

2. A Game with Complete Information

First consider a market for a homogeneous good with an incumbent, firm 1, and a potential entrant, firm 2. There are two periods, A and B. In period A, the incumbent with cost c_1 is the only firm in the market and faces an inverse market demand $p = 1 - q$. Initially, the incumbent chooses an output level in period A denoted q_1^A. At the end of period A, the potential entrant with constant marginal cost c_2 observes q_1^A and then decides whether to enter the market. In the absence of entry the incumbent remains a monopoly in period B. If firm 2 enters the market, the two firms play a Cournot game, i.e., quantity competition. In addition, the incumbent's second-period marginal cost decreases by an amount proportional to its first-period output, $c_1^B = c - \lambda q_1^A$, where the proportionality constant $\lambda > 0$ represents the speed of learning. There is an entry cost, $k > 0$, for the entrant. For simplicity, we assume that discount factor for values from period A to B is $\delta = 1$. In this paper subscripts denote firms ("1" or "2") and superscripts denote periods ("A" or "B") and market types ("M" for monopoly or "C" for Counot).

Note that since the incumbent can reduce its future cost through LBD, it would pick $q_1^{A*} = d_1/(2 - \lambda)$, where $d_i = 1 - c_i$ for $i = 1, 2$, instead of the static monopoly output level $d_1/2$, even without the threat of entry. We call this the dynamic monopoly output level. When the threat of another firm entering the market is present, profits for both firms depend on the first-period output of firm 1, q_1^A, and entry decision of firm 2 as follows.

If firm 2 does not enter the market, firm 1 remains a monopoly and earns profit:

$$\pi_1^{B,M} = \left[\left(d_1 + \lambda q_1^A \right)/2 \right]^2. \tag{1}$$

If firm 2 enters the market, the two firms earn Cournot profits:

$$\pi_1^{B,C} = \frac{(1 + c_2 - 2c_1 + 2\lambda q_1^A)^2}{9} \text{ and } \pi_2^{B,C} = \frac{(1 + c_1 - 2c_2 - \lambda q_1^A)^2}{9} - k . \tag{2}$$

The entry decision for firm 2 depends on the above calculations. It will enter if and only if $\pi_2^{B,C} > 0$. Note that firm 2 always enters when entry cost $k = 0$ since $\pi_2^{B,C}$ is nonnegative when $k = 0$. In this case, firm 1 chooses first-period output q_1^A to maximize total profits for both periods:

$$\pi_1(q_1^A) = \pi_1^A(q_1^A, c_1^A) + \pi_1^{B,C}(c_1^B(q_1^A), c_2) . \tag{3}$$

Then the optimal output level for firm 1 when firm 2 enters the market is:

$$q_1^{A**} = \frac{9(1 - c_1) + 4\lambda(1 + c_2 - 2c_1)}{18 - 8\lambda^2} .$$

Note that in the special case where the two firms have same initial cost, i.e., $c_1 = c_2 = c$, we have $q_1^{A**} > q_1^{A*} = d/(2 - \lambda)$ if $\lambda > 1/4$. So intuitively, the incumbent would raise the production over q_1^{A*} to build a cost advantage in the post-entry market provided it can ascend the learning curve fast enough. Expecting this q_1^{A**} from the incumbent, the entrant curtails its output, benefiting the incumbent.

Note also that the incumbent always has the option of deterring entry by picking a high enough first-period output level, i.e., there exists a $q_1^A(k)$ which makes $\pi_2^{B,C} = 0$ for $k > 0$. We call this the entry-deterring output level. The optimal choice of first-period output for firm 1, q_1^A, depends on value of $q_1^A(k)$ as follows.

- **Blocked entry:** If $q_1^A(k) < q_1^{A*}$, the incumbent operates as if there is no threat of entry. The first-period output level of firm 1 is the dynamic monopoly output level, q_1^{A*}, which sets its costs low enough to guarantee a loss to firm 2 if it enters. Firm 2 therefore does not enter the market. In this case, there is no real threat of entry.
- **Deterred entry:** If $q_1^{A*} < q_1^A(k) < q_1^{A**}$, the incumbent produces the lowest output that deters entry, $q_1^A(k)$, because its total profit decreases with q_1^A. In this case, the treat of entry drives the incumbent to overproduce in the pre-entry market.
- **Accommodated entry:** If $q_1^{A**} < q_1^A(k)$, the incumbent chooses the output level that maximizes total profit, i.e., firm 1 produces q_1^{A**} and accommodates entry if $\pi_1(q_1^A(k)) < \pi_1(q_1^{A**})$ but firm 1 produces $q_1^A(k)$ and deters entry if $\pi_1(q_1^A(k)) > \pi_1(q_1^{A**})$. In this case the incumbent raises its output under the threat of entry.

In summary, under complete information, unless the incumbent firm owns a much more superior production technique (i.e., much lower initial cost or rapid learning potential that can block potential entrants), the threat of entry pushes the incumbent firm to produce over its short-term monopoly output (lower price) to deter entry. When the entry cost is low, the incumbent firm might overproduce even

further to gain a cost advantage in the post-entry duopoly market. The loss in first-period profit is offset by the second-period gain from either remaining a monopoly or reducing the output from the entrant. A similar game is given in Tirole (1988) to illustrate how the incumbent can use LBD strategically to gain competitive advantage by overproducing in the first period. As we show in next section, overproducing (underpricing) might not be optimal for incumbent firms when it is also desirable to signal its cost type under information asymmetry.

3. A Game with Incomplete Information

Now consider the same game described above but with information asymmetry regarding the cost of the incumbent. More specifically, suppose that the incumbent firm knows its own cost, c_1, and the cost of the entrant firm, c_2, but the entrant does not know c_1 until it actually enters the market. The incumbent can reduce cost through LBD, and its learning curve is common knowledge to both firms. As a monopolist in the first period, the incumbent picks an output, q_1^A. The entrant observes this output (price) and then decides whether to enter or stay out. If it enters, firm 2 incurs an entry cost $k > 0$ (this is also common knowledge to both firms) and the two firms play a Cournot game. Otherwise, the incumbent remains a monopoly in the second period. We make the following assumptions in analyzing the game.

Assumption 1. The initial cost of firm 1, c_1, can be either high-type, denoted $\bar{c}_1$, or low-type, denoted $\underline{c}_1$. The prior probability distribution for c_1 is:

$$P(c_1 = \bar{c}_1) = x \text{ and } P(c_1 = \underline{c}_1) = 1 - x \text{ where } 0 < x < 1.$$

Assumption 2. Under complete information, $\pi_2(\underline{c}_1, c_2) < 0 < \pi_2(\bar{c}_2, c_2)$, i.e., the game ends up with blocked entry for low-cost firm 1 and accommodated entry for high-cost firm 1.

Assumption 2 keeps our focus on the more interesting cases, where the incumbent has a strong incentive to convey the information that it has a low cost, regardless of its true cost type. As discussed in Section 2, it is easy to verify that firm 1 prefers being a monopoly ($\pi_1^M(c_1, c_2) > \pi_1^C(c_1, c_2)$ for $c_1 = \bar{c}_1, \underline{c}_1$). Given assumption 2, the incumbent clearly wants to signal that it has low cost; however, it has no direct means of doing so even if it indeed has low cost. The indirect way is to signal by producing a low-cost quantity, $q_1^{A*}(\underline{c}_1)$, even when it has high cost. The loss from producing over the optimal quantity may be offset by the second-period gains from remaining a monopoly. Note that under mild assumptions both q_1^{A*} and q_1^{A**} are decreasing functions of c_1, i.e., $q_1^{A*}(\underline{c}_1) > q_1^{A*}(\bar{c}_1)$ and $q_1^{A**}(\underline{c}_1) > q_1^{A**}(\bar{c}_1)$. The low-cost incumbent tends to produce more (charge less) under complete information. To focus on the situation where learning is fast enough to be of strategic importance, we make the following assumption.

Assumption 3. Learning fast enough that firm 1's optimal pre-entry output given entry, q_1^{A**}, is higher than its dynamic monopoly output, q_1^{A*}.

As shown in Section 2, in the special case of same initial cost, the above assumption is equivalent to $\lambda > 1/4$, i.e., learning has to be faster than the threshold value $1/4$. If this assumption is violated, i.e., if $\lambda < 1/4$, learning is not a significant factor in deciding the pre-entry output.

The equilibrium analysis depends on the relative scale of the output choices by the incumbent of both types, which in turn depends on the model parameter values. Given the linear demand and learning structure, and the learning rate specified in Assumption 3, there are two possible orderings on the critical production levels:

- Case 1: $q_1^{A*}(\overline{c}_1) < q_1^{A**}(\overline{c}_1) < q_1^{A*}(\underline{c}_1) < q_1^{A**}(\underline{c}_1)$.
- Case 2: $q_1^{A*}(\overline{c}_1) < q_1^{A*}(\underline{c}_1) < q_1^{A**}(\overline{c}_1) < q_1^{A**}(\underline{c}_1)$.

We study pure strategy solutions in these two cases. In this game, a pure strategy for firm 1 is a mapping s_1 from the set of its possible costs, $\{\underline{c}_1, \overline{c}_1\}$, into the possible choice set of output $q_1^A \in \Re^+$. Note that the price and output have a one-to-one relationship, so the strategy s_1 can also be defined as a map from $\{\underline{c}_1, \overline{c}_1\}$ into the possible choice of $p_1^A \in \Re^+$, i.e., a higher output corresponds to a lower price. A pure strategy for firm 2 is a mapping s_2 from $\Re^{+2}$ into $\{0,1\}$, giving its decision for each possible pair $\{c_2, q_1^A\}$, where 0 represents "stay out" and 1 represents "enter".

As shown in Milgrom and Robert (1982), an appropriate method to analyze this dynamic game with incomplete information is to solve for a perfect Bayesian equilibrium (PBE). In the following we state the definition of PBE as defined in Fudenberg and Tirole (1992), with a little modification using our notation.

Definition 1. A PBE of a signaling game is a strategy profile (s_1^*, s_2^*) and posterior beliefs $\mu(c_1 | s_1)$ such that:

- $s_1^*(c_1) \in \arg\max_{s_1} \{\pi_1^A(s_1, s_2^*, c_1) + \pi_1^B(s_1, s_2^*, c_1)\}$ for all c_1,

- $s_2^*(s_1) \in \arg\max_{s_2} \sum_{c_1} \pi_2^B(s_1^*, s_2, c_1)\mu(c_1 | s_1^*)$ for all $s_1(c_1)$,

- $\mu(c_1 | s_1^*) = \dfrac{P(c_1)P(s_1 = s_1^* | c_1)}{\sum_{c_1} P(c_1)P(s_1^* | c_1)}$ if $\sum_{c_1} P(c_1)P(s_1^* | c_1) > 0$ and $\mu(c_1 | s_1^*)$ can

  be any probability distribution on $\{\underline{c}_1, \overline{c}_1\}$ if $\sum_{c_1} P(c_1)P(s_1^* | c_1) = 0$.

Potentially there are two types of equilibrium in this game: a separating equilibrium or a pooling equilibrium. In a separating equilibrium, the low-cost and the high-cost incumbents pick different first-period outputs (prices). Therefore the first-period price fully reveals the incumbent's cost type. In a pooling equilibrium, incumbents of both types choose the same first-period output (price). Therefore the potential entrant, observing the pre-entry output (price) of firm 1, cannot derive any information about the incumbent's cost type in addition to its original beliefs.

3.1 Case 1: Complementary Incentives

In this case, under complete information the low-cost incumbent firm would produce $q_1^{A*}(\underline{c}_1)$ more than the high-cost incumbent, which produces $q_1^{A**}(\overline{c}_1)$. Under incomplete information, both separating and pooling equilibria could exist, depending on the value of the entry-deterring output (price) of the high-cost incumbent, $q_1^A(k, \overline{c}_1)$. In the following proposition we identify a separating equilibrium and the sufficient conditions for its existence.

Proposition 1. Given Assumptions 1-3 and the ordering of output choices in Case 1, if the entry-deterring output for the high-cost firm 1, $q_1^A(k, \overline{c}_1)$, falls between $q_1^{A**}(\overline{c}_1)$ and $q_1^{A*}(\underline{c}_1)$, then there exists a separating equilibrium as follows:

$$s_1^*(\overline{c}_1, c_2) = q_1^{A**}(\overline{c}_1),$$
$$s_1^*(\underline{c}_1, c_2) = q_1^{A*}(\underline{c}_1),$$
$$s_2^*(q_1^A, c_2) = \begin{cases} \text{enter} & \text{if } s = q_1^{A**}(\overline{c}_1) \\ \text{do not enter} & \text{if } s = q_1^{A*}(\underline{c}_1). \end{cases} \tag{4}$$

Proof. We need to show that neither firm wants to deviate from the equilibrium strategy given in (4). Given that $q_1^{A**}(\overline{c}_1) < q_1^A(k, \overline{c}_1) < q_1^{A*}(\underline{c}_1)$, the high-cost firm 1 has no incentive to imitate a low-cost one because by imitating the high-cost firm 1 has to move from its optimal output level, $q_1^{A**}(\overline{c}_1)$ up to $q_1^{A*}(\underline{c}_1)$, which yields a lower profit even if this output deters entry completely. Therefore the high-cost incumbent, rather than limiting price, would let its pre-entry output level reveal its true type and accommodate entry in the second period. However, knowing that the high-cost firm would not imitate a low-cost one, the low-cost firm 1 would choose a (dynamic) monopoly output, $q_1^{A*}(\underline{c}_1)$, and deter entry. Firm 2 enters if $q_1^{A**}(\overline{c}_1)$ is observed and stays out if $q_1^{A*}(\underline{c}_1)$ is observed, i.e., the observed outputs fully reveal the incumbent's cost type and the game unfolds exactly the same way as one with complete information.

The above proposition states that, as the cost difference between the high-cost and low-cost incumbent grows large enough, at equilibrium the high-cost incumbent would charge an entry-accommodating price while the low-cost incumbent charges the monopoly (dynamic) price. On the other hand, if $q_1^A(k, \overline{c}_1) > q_1^{A*}(\underline{c}_1)$, we can expect that there exists a pooling equilibrium where incumbents of both types produce $q_1^{A*}(\underline{c}_1)$. The existence of this pooling equilibria hinges on a posterior probability distribution on cost types that satisfies:

$$\pi_2^B(\overline{c}_1, q_1^A(\underline{c}_1), c_2)x + \pi_2^B(\underline{c}_1, q_1^A(\underline{c}_1), c_2)(1-x) \le 0, \tag{5}$$

i.e., the expected profit for firm 2 is negative when firm 2 cannot derive information on the incumbent's cost beyond the prior probability.

As illustrated above, in the case of complying incentives, the high-cost incumbent has incentives to raise output to imitate a low-cost firm and the low-cost incumbent has incentives to raise output to distinguish itself from the high-cost firm. In summary, LBD and the information asymmetry push the pre-entry output in the same direction, i.e., they result in a pre-entry output level greater than or equal to the output choice in the game in which only one is of strategic consideration. Consequently, social welfare is higher than under complete information.

3.2 Case 2: Conflicting Incentives

In this case the entry-accommodating output for the high-cost incumbent is higher than the dynamic monopoly output for the low-cost incumbent. Intuitively, this means that under complete information the high-cost firm has to produce more aggressively (sacrifice more profit), compared to the low-cost firm in the pre-entry period to gain a cost advantage in the post-entry market. Under information asymmetry, however, the high-cost firm has an additional option of hiding its true cost type by imitating the low-cost incumbent, i.e., by producing less, provided this strategy can reduce or deter entry. A low-cost incumbent, on the other hand, may desire to reduce production to reveal its cost type. Therefore the output choices at equilibrium, as a result of balancing these conflicting incentives, are likely to be lower than that in a game with complete information. In the following propositions we present both a separating equilibrium and a pooling equilibrium and the sufficient conditions for their existence.

Proposition 2. Given Assumptions 1-3 and the ordering of output choices in Case 2, there exists a separating equilibrium as follows:

$$s_1^*(\overline{c}_1, c_2) = q_1^{A**}(\overline{c}_1),$$
$$s_1^*(\underline{c}_1, c_2) = q_1^{A***}(\underline{c}_1),$$
$$s_2^*(q_1^A, c_2) = \begin{cases} \text{enter} & \text{if} \quad s = q_1^{A**}(\overline{c}_1) \\ \text{do not enter} & \text{if} \quad s = q_1^{A***}(\underline{c}_1), \end{cases} \tag{6}$$

where $q_1^{A***}(\underline{c}_1) < q_1^{A*}(\underline{c}_1)$ if the following conditions hold:

$$\pi_1^A\left(\overline{c}_1, q_1^{A**}(\overline{c}_1)\right) + \pi_1^{B,C}\left(\overline{c}_1 - \lambda q_1^{A**}(\overline{c}_1), c_2\right) \geq \pi_1^A\left(\overline{c}_1, s_1^*(\underline{c}_1)\right) + \pi_1^{B,M}\left(\overline{c}_1 - \lambda s^*(\underline{c}_1), c_2\right), \tag{7}$$

$$\pi_1^A\left(\underline{c}_1, s_1^*(\underline{c}_1)\right) + \pi_1^{B,M}\left(\underline{c}_1 - \lambda s^*(\underline{c}_1), c_2\right) \geq \pi_1^A\left(\underline{c}_1, q_1^{A*}(\underline{c}_1)\right) + \pi_1^{B,C}\left(\underline{c}_1 - \lambda q_1^{A*}(\underline{c}_1), c_2\right). \tag{8}$$

Proof. To show that the strategies in (6) are in equilibrium, we need to verify that the high-cost firm 1 does not want to imitate the low-cost firm's equilibrium output, $s_1^*(\underline{c}_1)$, and vice versa. Note that in a separating equilibrium, the output of the incumbent reveals its true cost type, so a high-cost firm producing $s_1^*(\overline{c}_1) = q_1^{A**}(\overline{c}_1)$ would get payoff $\pi_1^A(\overline{c}_1, q_1^{A**}(\overline{c}_1)) + \pi_1^{B,C}(\overline{c}_1 - \lambda q_1^{A**}(\overline{c}_1), c_2)$. On the other hand, if the high-cost firm 1 deviates from equilibrium and produces output of the low-cost firm,

$s^*(\underline{c}_1)$, it can at best completely deter entry and get profit $\pi_1^A(\overline{c}_1, s_1^*(\underline{c}_1)) + \pi_1^{B,M}(\overline{c}_1 - \lambda s_1^*(\underline{c}_1), c_2)$. Condition (7) makes it undesirable for the high-cost firm to imitate a low-cost firm. Similarly, given condition (8), the low-cost firm gets higher total profit by over-producing in the pre-entry period to distinguish itself as a low-cost firm than being a monopoly in pre-entry market and taking the risk of being mistaken as a high-cost firm.

Proposition 2 states that in a separating equilibrium, $s_1^*(\underline{c}_1) \neq s_1^*(\overline{c}_1)$, firm 2 enters if $s_1^*(\overline{c}_1)$ is observed and stays out if $s_1^*(\underline{c}_1)$ is observed, i.e., "…entry takes place in exactly the same circumstances as if the entrant had been informed about the value c_1, i.e., with probability p" (Milgrom and Roberts, 1982). In this case, the pre-entry output cannot be used to reduce the entry probability relative to the complete information case. However, the low-cost firm 1 still has incentives to reduce output (raise price) from its dynamic monopoly choice, $q_1^{A*}(\underline{c}_1)$, to distinguish itself from the high-cost firm. The social welfare is lower than under complete information. The first-period welfare is generally decreased because the low-cost firm 1 lowers its output. The second-period welfare is also lower because the chance of entry is the same, but the low-cost firm 1 has an inflated cost due to under-production in the first period.

A pooling equilibrium can also exist as in the following proposition.

Proposition 3. Given Assumptions 1-3 and the ordering of output choices in Case 2, there exists a pooling equilibrium as follows:

$$s_1^*(\underline{c}_1, c_2) = s_1^*(\overline{c}_1, c_2) = q_1^{A*}(\underline{c}_1) ,$$

$$s_2^*(c_2, q_1^A) = \begin{cases} 0 & \text{if} \quad s_1 = q_1^{A*}(\underline{c}_1) \\ 1 & \text{if} \quad s_1 \neq q_1^{A*}(\underline{c}_1), \end{cases} \tag{9}$$

as long as the prior probability distribution on the cost types of firm 1 is such that:

$$(1-x)\pi_2^{B,C}\left(\underline{c}_1 - \lambda s_1^*, c_2\right) + x\pi_2^{B,C}\left(\overline{c}_1 - \lambda s_1^*, c_2\right) \leq 0 , \tag{10}$$

and for the incumbent firm:

$$\pi_1^A\left(\overline{c}_1, q_1^{A*}(\underline{c}_1)\right) + \pi_1^{B,M}\left(\overline{c}_1 - \lambda q_1^{A*}(\underline{c}_1), c_2\right) \geq \pi_1^A\left(\overline{c}_1, q_1^{A**}(\overline{c}_1)\right) + \pi_1^{B,C}\left(\overline{c}_1 - \lambda q_1^{A**}(\overline{c}_1), c_2\right). \tag{11}$$

Proof. At a pooling equilibrium, the potential entrant can infer no information from the pre-entry output level, so its beliefs about the distribution on the cost type of firm 1 is the same as the prior probability distribution, i.e., $P(c_1 = \overline{c}_1) = x$ and $P(c_1 = \underline{c}_1) = 1 - x$. Given an equilibrium pre-entry output s_1^* and the prior probability, firm 2 calculates its expected profit from entering. It would enter if the expected profit is positive, otherwise it would stay out. Condition (10) specifies a prior probability that yields negative expected profit for the potential entrant in the post-entry period. Given a pooling strategy of firm 1, this condition imposes an equilibrium strategy for the entrant to stay out.

For the incumbent, it is easy to check that $s_1^*(\underline{c}_1) = q_1^{A*}(\underline{c}_1)$ is optimal for the low-cost firm 1 since any divergence from $q_1^{A*}(\underline{c}_1)$ would both induce entry and reduce profits. Given condition (11), the high-type firm 1 gets higher total profit by producing $q_1^{A*}(\underline{c}_1)$ rather than its own entry-accommodating output. The right-hand-side of the inequality represents the best profit the high-type firm 1 can get by diverting from the equilibrium output level. The left-hand-side of the inequality represents the profit for the high-type firm 1 by imitating the low-type firm's optimal output $q_1^{A*}(\underline{c}_1)$. Therefore under condition (11) it is desirable for a high-cost firm 1 to stick with $s_1^*(\overline{c}_1, c_2)$.

Proposition 3 states that when the prior probability distribution regarding the cost types of the incumbent yields a negative expected profit for the entrant, there exists a pooling strategy. The low-cost incumbent produces its dynamic monopoly output while the high-cost incumbent reduces its first period output (raises price) to imitate a low-cost firm. The pre-entry market price/output does not reveal cost type of the incumbent firm and entry is deterred. For this reason, the social welfare is lower than a situation with complete information.

The key factor driving this result is that under complete information a high-cost incumbent would produce more than a low-cost incumbent, i.e., $q_1^{A**}(\overline{c}_1) > q_1^{A*}(\underline{c}_1)$. It is natural to ask what learning rate, λ, supports this reversed output choices by incumbent firms with different initial costs? It is hard to make a general statement but given the particular functions in our model, it is not hard to see that this is possible. From the previous discussion we know that in a special case of $\underline{c}_1 = \overline{c}_1 = c_2 = c$, this relationship $q_1^{A**} > q_1^{A*}$ holds when $\lambda < 1/4$. The difference between q_1^{A**} and q_1^{A*} increases with λ, i.e., the faster learning occurs the more a high-cost firm 1 would over-produce. In addition, both quantities are decreasing functions in c_1 when $\lambda < 3/2$. Therefore it can be shown that as $\underline{c}_1 \neq \overline{c}_1$ there exists a λ so that $q_1^{A**}(\overline{c}_1) > q_1^{A*}(\underline{c}_1)$, as long as the difference between the two cost types, $\underline{c}_1$ and $\overline{c}_1$, is not too large.

Milgrom and Roberts (1982) argue that limit-pricing emerges naturally in the world of asymmetric information and it actually increases social welfare. In case 2 of our model, however, the incumbent firm actually distorts its price upward (output downward) to deter entry. This higher market price leads to lower overall social welfare. Though it would be premature to conclude that asymmetric information always decreases welfare, the propositions above provide counterexamples of limit pricing. We might call this behavior anti-limit pricing.

4. Concluding Remarks

It is well known that in a game with complete information, the threat of entry makes it desirable for the incumbent firm to raise output to gain cost advantage through LBD. This strategy leads to a market price lower than the optimal (dynamic) monopoly price and therefore raises social welfare in the market. We show in this paper, however, that when information is asymmetric, the firm's incentive to signal its cost type might hurt the incentive to lower cost by over-producing. As a

consequence the incumbent may charge a price higher than it would in an environment with complete information, therefore reducing the social welfare.

This impact of information asymmetry on a firm's pricing strategy differs from the classic limit-pricing model (Milgrom and Roberts, 1982). Milgrom and Roberts (1982) argue that with information asymmetry, "limit pricing, or more generally, deviations from short run maximizing behavior, then emerges in equilibrium." In our model, firms deviate from their short-term maximizing behavior as well, but instead of "limit pricing" they "limit production," leading to a higher price. The reason for this different result from information asymmetry is that in Milgrom and Roberts (1982) learning does not occur with production, so real profitability of the post-entry market does not change with the pre-entry price, whereas in our model the incumbent firm's ability to change the prospective profitability of the post-entry market through learning enriches its choice set. This enrichment of choices and information asymmetry relieves the pressure on incumbents to reduce cost. In this sense our model provides a new angle of understanding how information asymmetry affects the firm's strategic pricing behavior to under threat of entry.

We use linear functions and a two-state cost structure to simplify the analysis. The conclusion can be generalized if we could use more general functional forms to describe the market demand, the learning process, and the incumbent's cost probability distribution. Further research may also extend the model to more than two periods, as firm strategies are generally different under a longer time horizon.

Appendix

Table 1. Critical Values in a Two-Period Game under Complete Information

Description	Period A	Period B
Firm 1 marginal cost	c_1	$c_1 - \lambda q_1^A$
Market demand	$p = 1 - q_1^A$	$p = 1 - q_1^B - q_2^B$
Firm 2 profit without entry	$\pi_1^A = q_1^A(1 - c_1 - q_1^A)$	$\pi_1^{B,M} = (1 - c_1 + \lambda q_1^A/2)^2$
Firm 1 profit with entry	$\pi_1^A = q_1^A(1 - c_1 - q_1^A)$	$\pi_1^{B,C} = (1 + c_2 - 2c_1 + 2\lambda q_1^A)^2/9$
Firm 2 profit with entry		$\pi_2^{B,C} = [(1 + c_1 - 2c_2 - \lambda q_1^A)^2/9] - k$
Firm 1 optimal q without entry	$q_1^{A*} = 1 - c_1/2 - \lambda$	$q_1^{B*} = 1 - c_1/2 - \lambda$
Firm 1 optimal q with entry	$q_1^{A**} = 9(1 - c_1) + 4\lambda(1 + c_2 - 2c_1)/18 - 8\lambda^2$	
Firm 1 entry deterring quantity	$q_1^A(k, c_1) = 1 + c_1 - 2c_2 - 3\sqrt{k}/\lambda$	

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