A Fast Mixed Integer Programming Formulation for the Maximum Set Covers Problem in Wireless Sensor Networks

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Abstract*-In a wireless sensor network (WSNET), the target coverage (TC) problem is to schedule the activity of each sensor such that each target is monitored by some sensor at every moment and the network lifetime is maximized. A possible approach to deal with the TC problem is to organize all the sensors into a group of non-disjoint sets such that each set can completely monitor all the targets within a certain time interval and only one set is active at any time instant. This approach is known as the maximum set covers (MSC) problem, which has been proven to be NP-complete. In this paper, the MSC problem is studied. There has existed a mixed integer programming formulation (MIPF) for the MSC problem, named as MIPF-for-MSC, which can find its optimal solution. However, the execution time of MIPF-for-MSC is heavy. In this paper, we design a preprocessing technique and a new inequality to speed up the execution of MIPF-for-MSC. Computer simulations show that compared with the original MIPF-for-MSC, our preprocessing technique and new inequality can reduce the execution time significantly.*

Keywords: integer programming, maximum set cover, power-saving, target coverage, wireless sensor network.

1. Introduction

A WSNET is formed by a large number of tiny sensing devices (or called *sensors*) [10] [12]. A sensor in a WSNET can generate as well as forward data, which are gathered from every sensor's vicinity and will be delivered to the single remote base station (or called the *sink*). Two sensors in such a network can communicate directly with each other through a single-hop routing path in the shared wireless media if their positions are close enough. Otherwise, they need a multi-hop routing path to carry out their communications. In a multi-hop routing path, the data packets sent by a source sensor are relayed by several intermediate sensors before reaching the sink. WSNETs are useful in a broad range of environmental sensing applications such as vehicle tracking, seismic data, and so on.

Since WSNETs are characterized by their limited battery-supplied power, the network lifetime is restricted at the battery power and the speed of power-consumption of each sensor. Extensive research efforts have been devoted to the design of power-saving mechanisms such that the total power consumption in a WSNET is minimized and the network lifetime is maximized. In this paper, the lifetime of a WSNET is defined to be the time period from the beginning of the network operation to when one of the targets can not be monitored. A possible power-saving mechanism is to schedule each sensor to alternate its states between the active and sleep mode. This is because while some requirements are met, compared with another case of each sensor being active continuously, the case of each sensor altering its states between the active and sleep mode will generate a longer network lifetime [1-3].

One of the most important design issues in a WSNET is the TC problem [2]. In the TC problem, m targets are located in known locations. Given a WSNET consisted of n sensors, where these sensors are randomly distributed near by these m targets such that a sensor can monitor one or some targets, the TC problem is to schedule the activity of each sensor such that each target is monitored by at least one sensor at every moment and the network lifetime is maximized. The TC problem has attracted a lot of attention recently [1-3]. In particular, a possible approach to deal with the TC problem is to organize all the sensors into a group of non-disjoint sets such that each set can completely monitor all the targets during a certain time interval and only one set is active at any time instant. In other words, these sensor sets in this group are activated successively. At any time instance, each sensor belonging to the active set is in its active state while all the other sensors are in the sleep state. This approach is known as the MSC problem, which has been proven to be NP-complete [2].

In this paper, the MSC problem is studied. Mixed integer linear programming formulations (MILPFs) [7] have been adopted by many researchers to solve various problems in wireless networks [4-6] [11]. Similarly, there has existed a mixed integer programming formulation (MIPF) for the MSC problem, named as MIPF-for-MSC, which can find its optimal solution [2]. However, the execution However, the execution time of MIPF-for-MSC is very long. Several efficient schemes have been proposed to speed up the execution of a MIPF [9]. For example, a preprocessing technique can be applied to the given input before the execution of a MIPF. Another efficient scheme is to add more inequalities to the original MIPF. In this paper, an efficient preprocessing technique and an efficient inequality are proposed to speed up the execution of MIPF-for-MSC (i.e., to speed up the finding of solutions to the MSC problem). Simulation results show that compared with the original MIPF-for-MSC, our preprocessing technique and new inequality can reduce the execution time significantly.

The rest of the paper is organized as follows. In Section 2, a formal definition of the MSC problem is given. In Section 3, the existing MIPF-for-MSC is presented. In Section 4, an efficient preprocessing technique and an efficient inequality for the MIPF-for-MSC are proposed. In Section 5, the performance of the proposed preprocessing technique and inequality is evaluated through computer simulations and compared to that of the original MIPF-for-MSC. Lastly, Section 6 concludes the whole research.

2. Problem Definition

In this section, some assumptions and notations for the MSC problem are given first. Then, the MSC problem is defined formally and explained in detail [2].

Assumptions and Notations for the MSC Problem [2]

The following states some important assumptions and notations used in the MSC problem considered in this paper.

(1) Every sensor has the same sensing range. The sensing range of a sensor is centralized in itself. It may monitor all the targets within the area of its sensing radius.

(2) Every sensor has the same battery power. The lifetime of every sensor is defined to be one time unit.

(3) If the sensing range of a sensor is larger than the distance between itself and a target, then the sensor can monitor the target. If a target is out of the sensing range of a sensor, then the target can not be monitored by the sensor.

(4) The state of a sensor is either active or sleep.

(5) There are m targets $v_{r_1}, v_{r_2}, \cdots, v_{r_m}$ to be monitored. There are n sensors $v_{s_1}, v_{s_2}, \cdots, v_{s_n}$ in a WSNET. These sensors are randomly deployed to monitor all the targets. That is, each target is required to be always monitored by at least one sensor at any time.

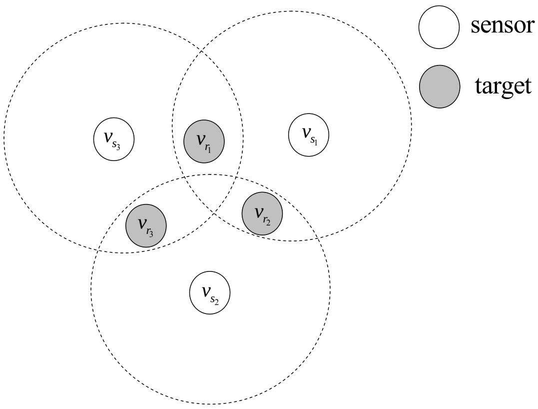
Definition of the MSC Problem

In a given WSNET, all the targets must be monitored by one or more sensors at any time. A group of sensors is called a *cover set* S_k if all the targets in the WSNET can be monitored by the sensors in S_k during a time interval of length t_k. The parameter t_k is named as the *time weight* associated with S_k. For a given group of cover sets $U = \{S_k \mid k = 1, 2, \cdots, p\}$, the calculation of each t_k is as follows. Let $S_k = \left\{ v_{s_{k_1}}, v_{s_{k_2}}, \cdots, v_{s_{k_n}} \right\}$. If there are x cover sets each of which includes $v_{s_{k_i}}$, then the lifetime of $v_{s_{k_i}}$ is $\hat{v}_{s_{k_i}} = 1/x$. Thus, $t_k = \min_{v_{s_{k_i}} \in S_k} \hat{v}_{s_{k_i}}$.

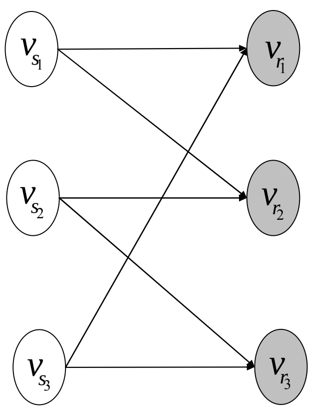
Now the MSC problem is defined formally as follows. Given a WSNET consisting of a set of targets $R = \{v_{r_j} \mid j = 1, 2, \cdots, m\}$ and a set of sensors $C = \{v_{s_i} \mid i = 1, 2, \cdots, n\}$, find a group of cover sets $U = \{S_k \mid k = 1, 2, \cdots, p\}$ in which every cover set S_k has a time weight t_k such that the summation of time weights, $T = t_1 + t_2 + \cdots + t_p$, is maximized, where the value of t_k in $[0,1]$. To be more specific, the MSC problem is to find a group of cover sets $S_1, S_2, \cdots, S_p$ such that all the targets are continually monitored by each cover set S_k during a time interval of length t_k and the network lifetime $t_1 + t_2 + \cdots + t_p$ is maximized.

As an illustration of the above notations and definitions, let us consider the following example. Figures 1 and 2 show an instance of the MSC problem. Figure 1(a) shows a WSNET consisting of three sensors $v_{s_1}, v_{s_2}, v_{s_3}$ and three targets $v_{r_1}, v_{r_2}, v_{r_3}$. All of the three sensors $v_{s_1}, v_{s_2}, v_{s_3}$ have the same sensing radius. Figure 1(b) represents the relationship between the sensors and the targets in Figure 1(a). An arrow from a sensor v_{s_i} to a target v_{r_j} denotes that target v_{r_j} can be monitored by sensor v_{s_i}. For example, there exist an arrow between targets v_{s_1} and v_{r_1}/v_{r_2}. This indicates that targets v_{r_1} and v_{r_2} can be monitored by sensor v_{s_1}. Similarly, targets v_{r_2} and v_{r_3} can be monitored by sensor v_{s_2}. Targets v_{r_1} and v_{r_3} can be monitored by sensor v_{s_3}. Figure 2 shows a group of possible cover sets for the MSC problem defined by Figure 1. Figure 2(a) shows that all the three targets $v_{r_1}, v_{r_2}, v_{r_3}$ can be monitored by sensors v_{s_2} and v_{s_3} simultaneously during the first time interval of length t_1. That is, $S_1 = \{v_{s_2}, v_{s_3}\}$. Similarly, Figure 2(b) shows that all the targets can be monitored by sensors v_{s_1} and v_{s_2} simultaneously during the second time interval of length t_2, i.e., $S_2 = \{v_{s_1}, v_{s_2}\}$. Finally, Figure 2(c) shows $S_3 = \{v_{s_1}, v_{s_3}\}$ during the third time interval of length t_3. To sum up, all the three targets $v_{r_1}, v_{r_2}, v_{r_3}$ can be completely monitored by the sensors in S_1, S_2, and S_3, respectively, during three different time intervals.

As each sensor is used to monitor the targets twice in the three different time intervals

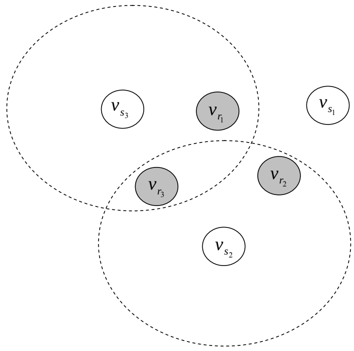


(a) A WSNET.

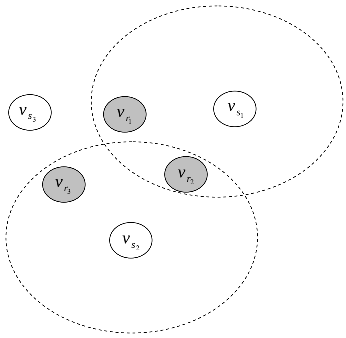


(b) The relationship of coverage between sensors and targets.

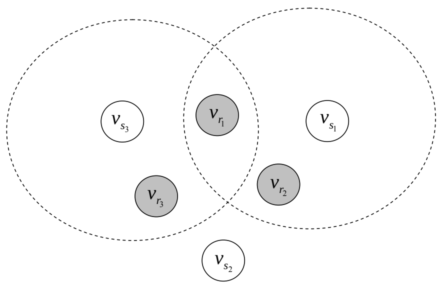
Figure 1. An instance of the MSC problem.



(a) $S_1 = \{v_{s_2}, v_{s_3}\}$.



(b) $S_2 = \{v_{s_1}, v_{s_2}\}$.



(c) $S_3 = \{v_{s_1}, v_{s_3}\}$.

Figure 2. A group of possible cover sets for the MSC problem in Figure 1.

(i.e., each sensor appears twice in the three different cover sets, S_j, $j = 1,2,3$), the time weight t_i of each cover set S_i is 0.5. Thus, the network lifetime of the WSNET given in Figure 1 is $t_1 + t_2 + t_3 = 1.5$.

3. An existing MIPF for the MSC problem

In this section, an existing MIPF for the MSC problem, named as MIPF-for-MSC, proposed in [2] is presented.

Network Model

A WSNET is represented by a finite set of sensors $C = \{v_{s_i} \mid i = 1,2,\cdots,n\}$ and a set of targets $R = \{v_{r_j} \mid j = 1,2,\cdots,m\}$. A set of $C_k = \left\{ i \mid sensor\ v_{s_i}\ monitors\ target\ v_{r_k} \right\}$ is used to describe the relationship between sensors and targets. That is, if sensor v_{s_i} can monitor target v_{r_k}, then i is put into C_k.

A Known MIPF for optimally solving the MSC problem: MIPF-for-MSC

The variables used in MIPF-for-MSC are defined in the following. x_{ij}: a binary variable, where $i = 1,2,\cdots,n$ and $j = 1,2,\cdots,p$. Its value is 1 when sensor $v_{s_i} \in S_j$ and 0 otherwise, where S_j is a cover set. t_j: the time weight of cover set S_j, where $j = 1,2,\cdots,p$. Its value is between 0 and 1.

Thus, MIPF-for-MSC can be described as follows:

Maximize:

$$t_1 + t_2 + \cdots + t_p \quad (1)$$

Subject to:

$$\sum_{j=1}^{p} x_{ij} t_j \le 1 \quad \text{for all } v_{s_i} \in C \quad (2)$$

$$\sum_{i \in C_k} x_{ij} \ge 1 \quad \text{for all } v_{r_k} \in R, j = 1,2,\cdots,p \quad (3)$$

$$x_{ij} = 0 \text{ or } 1,\ x_{ij} = 1 \text{ if and only if } v_{s_i} \in S_j \quad (4)$$

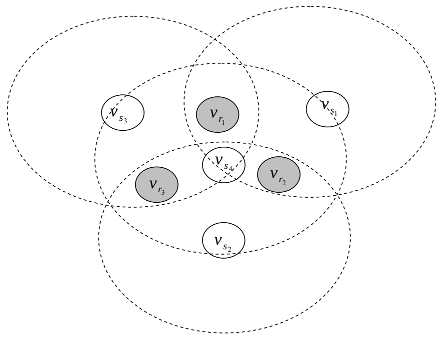
The objective function (1) is used to maximize the network lifetime. The inequality (2) states that the total time interval scheduled for each sensor in all set covers is not larger than 1, which is the lifetime of each sensor. Inequality (3) guarantees that every target v_{r_k} is monitored by at least one sensor v_{s_i} in every cover set S_j. Inequality (4) expresses the integrality of variable x_{ij}

4. Our Efficient Preprocessing Technique and Inequality

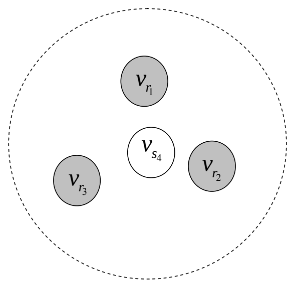
In this section, we propose an efficient preprocessing technique and a new inequality to speed up the execution of MIPF-for-MSC.

Our Preprocessing Technique

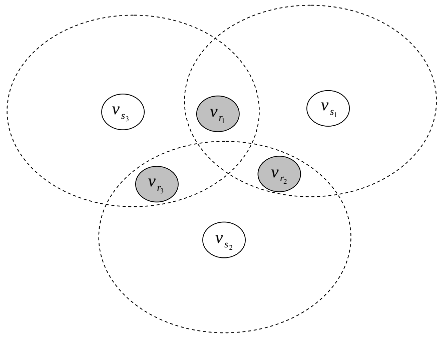
First, let us use an example to explain the idea behind our preprocessing technique. Consider Figure 3(a). The locations of all the three targets are within the sensing radius of sensor v_{s_4}. Thus, sensor v_{s_4} can monitor targets v_{r_1}, v_{r_2}, v_{r_3} simultaneously, as shown by Figure 3(b). Therefore, it is feasible to schedule sensor v_{s_4} to monitor all the targets by itself in a single time interval. Figure 3(c) shows



(a) A WSNET with four sensors.



(b) Targets $v_{r_1}, v_{r_2}, v_{r_3}$ can be monitored by sensor v_{s_4} simultaneously.



(c) The simplified WSNET.

Figure 3. An example to illustrate our preprocessing technique.

the resultant WSNET after sensor v_{s_4} is deleted. Obviously, Figure 3(c) is the same as Figure 1. In other words, Figure 3(a) can be simplified to become Figure 1 before the MIPF-for-MSC is applied to it.

Based on the observation, our idea is that a sensor may be deleted from the WSNET before the execution of the MIPF-for-MSC if it can monitor all the targets by itself in a single time interval. The computer simulations in Section 5 show that such deletions, i.e., such a preprocessing technique, can indeed speed up the execution of MIPF-for-MSC.

Our New Inequality

First, let us explain the idea behind our new inequality via an example. Consider Figure 4. Target v_{r_1} can be monitored by at most two sensors: v_{s_1} and v_{s_3}. Target v_{r_3} can be monitored by at most two sensors: v_{s_2} and v_{s_3}. Target v_{r_2} can be monitored by at most three sensors: v_{s_1}, v_{s_2}, and v_{s_4}. Thus, it is not hard to discover that the network lifetime of of the WSNET in Figure 4 can not exceed 2. In other words, the network lifetime $\sum_{j=1}^{p} t_j$ of a WSNET is dominated by the minimum T_c among the maximum numbers of sensors which can monitor a certain target. As a result, we have a new inequality $\sum_{j=1}^{p} t_j \le T_c$. In Figure 4, it canbe observed that T_c is equal to 2. Hence, $\sum_{j=1}^{p} t_j \le 2$. Furthermore, if there exist M_s sensors each of which can monitor all the targets in the WSNET by itself, then our new inequality

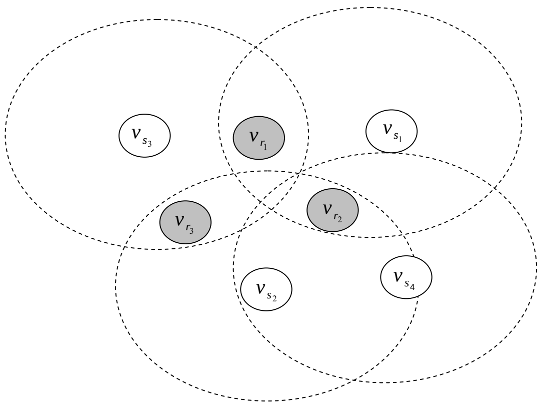


Figure 4. An example to illustrate our new inequality (9).

can be rewritten as $M_s \le \sum_{j=1}^{p} t_j \le T_c$.

Obviously, we must calculate the values of M_s and T_c for a given WSNET before our new inequality can be applied. Our computer simulations show that the time to find the values of M_s and T_c is very short.

Now, our new MIPF for the MSC problem, named as NMIPF-for-MSC can be described as follows:

Maximize:

$$t_1 + t_2 + \cdots + t_p \tag{5}$$

Subject to:

$$\sum_{j=1}^{p} x_{ij} t_j \le 1 \quad \text{for all } v_{s_i} \in C \tag{6}$$

$$\sum_{i \in C_k} x_{ij} \ge 1 \quad \text{for all } v_{r_k} \in R, j = 1, \cdots, p \tag{7}$$

$$x_{ij} = 0 \text{ or } 1, \quad x_{ij} = 1 \text{ if and only if } v_{s_i} \in S_j \tag{8}$$

$$M_s \le \sum_{j=1}^{p} t_j \le T_c \tag{9}$$

5. Computer Simulations

In this section, we examine the efficiency of our preprocessing technique and new inequality through computer simulations. Our performance comparisons are conducted among the three different formulations: (1) The original MIPF for the MSC problem: MIPF-for-MSC, which consists of inequalities (1) to (4). (2) Our new MIPF for the MSC problem: NMIPF-for-MSC, which consists of inequalities (5) to (9). (3) Our NMIPF-for-MSC + our preprocessing technique. The three formulations are solved by the LINGO 8.0 software package [8] run at a typical personal computer consisted of Intel Core 2 Duo 2.13 GHz and 1G MB DDRII SDRAM. The execution time of each formulation is observed.

Our computer simulations are carried out on a number of WSNETs generated randomly. The sensors and targets are randomly located on a grid of $500m \times 500m$. Every sensor has the same sensing radius, which is set to $250m$. Our computer simulations consider two different cases. In case 1, it is assumed that there exist sensors which can monitor all the targets by itself at the same time. There do not exist such sensors in case 2. In other words, only case 1 has cover sets consisting of a single sensor.

For case 1, the execution times required by each of the three different formulations are shown in Table 1, where 3600s↑ denotes that the execution time exceeds 3600 seconds. Table 1 shows that compared with MIPF-for-MSC, our NMIPF-for-MSC is able to shorten the execution times from $(4.63-1.00)/4.63=78.40\%$ to $(1151.86-511.93)/1151.86=55.56\%$ when the network size is from $|C|=15/|R|=5$ to $|C|=30/|R|=5$. Moreover, our NMIPF-for-MSC + our preprocessing technique can shorten the execution times from $(4.63-0.88)/4.63=80.99\%$ to $(1151.86-231.79)/1151.86=79.88\%$ when the network size is from $|C|=15/|R|=5$ to $|C|=30/|R|=5$. To sum up, the execution times of our NMIPF-for-MSC and our NMIPF-for-MSC + our preprocessing technique are clearly much less than that of MIPF-for-MSC.

For case 2, the simulation results are shown in Table 2. Compared with MIPF-for-MSC, our NMIPF-for-MSC can shorten the execution time up to $(44.82-25.73)/44.82=42.59\%$ and $(616.70-156.55)/616.70=74.61\%$ when the network size is $|C|=20/|R|=5$ and $|C|=25/|R|=5$, respectively. It can be observed that our new inequality is able to shorten the execution time in most cases.

6. Conclusions

In this paper, we have studied the MSC problem in WSNETs. The MSC problem has been proven to be NP-complete and a MIPF for its optimal solutions has been proposed. However, the existing MIPF has a heavy execution time. This makes the finding of the optimal solutions of the MSC problem impractical in most situations. In this paper, we have designed an efficient preprocessing technique and an efficient inequality to speed up the execution of the existing MIPF. The computer simulations verify that compared with the original MIPF, our preprocessing technique and inequality can reduce the execution time in most cases. In particular, when there exist sensors which can monitor all the targets in the WSNET by itself at the same time, the reduction is significant.

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Table 1. The execution times of different MIPFs with WSNETs including single-sensor cover sets.

execution time (seconds) \ |C| / |R|; different MIPFs for the MSC problem	|C|=15 /|R|=5	|C|=20 /|R|=5	|C|=25 /|R|=5	|C|=30 /|R|=5	|C|=35 /|R|=5	|C|=40 /|R|=5
MIPF-for-MSC	4.63s	403.00s	910.88s	1151.86s	3600s↑	3600s↑
NMIPF-for-MSC	1.00s	8.33s	72.63s	511.93s	921.92s	3600s↑
NMIPF-for-MSC + our preprocessing technique	0.88s	10.08s	70.25s	231.79s	516.50s	1074.33s

Table 2. The execution times of different MIPFs with WSNETs without single-sensor cover sets.

execution time (seconds) \ |C| / |R|; different MIPFs for the MSC problem	|C|=15 /|R|=5	|C|=20 /|R|=5	|C|=25 /|R|=5	|C|=30 /|R|=5	|C|=35 /|R|=5	|C|=40 /|R|=5
MIPF-for-MSC	0.90s	44.82s	616.70s	330.65s	3600s↑	3600s↑
NMIPF-for-MSC	1.75s	25.73s	156.55s	423.40s	825.10s	3600s↑

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