Blending Operations with Blending Range Controls on Primitives' Subsequent Blends

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Abstract- *In implicit surfaces, most existing blends keep their primitives' properties unchanged on non-blending regions, like pure Boolean set operations* $Max/Min(f_1,\ldots,f_k)$. *Hence, when they are used as a new primitive in other blends, their primitives always have similar subsequent blending surfaces with the other primitives in their subsequent blends. To solve this problem, this paper proposes new blending operations that can provide parameters to individually adjust their primitives' subsequent blending surfaces, without deforming their original blending surfaces. The newly proposed blending operations provide parameters* $m_1,\ldots,m_k$ *to enable their primitives* $f_1,\ldots,f_k$ *to behave like* $Max/Min(f_1/m_1,\ldots,f_k/m_k)$ *on non-blending regions and its original blending surface does not change whatever positive values* $m_1,\ldots,m_k$ *are set. As a result, the subsequent blending surfaces of their primitives* $f_1,\ldots,f_k$ *can be adjusted respectively by varying* $m_1,\ldots,m_k$. *Furthermore, this paper proposes a generalized method that can transform some of existing blending operators into the proposed blending operations stated above.*

Keywords: Implicit surfaces, Boolean set operations, The displacement method, The scale method.

1. Introduction

Implicit surface modeling is attracting much attention because a complex implicitly defined object can be constructed freely from some simple primitives, such as planes, sphere, cone,…,etc., via a successive composition of blending operations, such as Boolean set operations. In implicit surfaces, blending operations play a very important role because they can connect intersecting objects with automatically generated transitions to smooth out the unwanted sharp edges, kinks, and creases. In the previous literature, R-functions [17] give Boolean set operations with C^n, n≥1, continuity. In addition, blends that can provide blending range parameters to adjust the size of the resulting blending surface and to limit the blending surface in specific regions without deforming the overall shapes of blended primitives can be found in [3, 4, 5, 8, 9, 10, 11, 12, 13, 16, 18, 20, 21, 22, 23]. For low-degree computing complexity, soft objects modeling were also proposed. It defined primitives, called soft objects, using field functions, which were proposed in [1, 2, 6, 7, 12, 14, 15, 24, 25]. Due to the use of field functions, soft objects can be blended easily by performing addition operations only. Besides, some other blending operators that can offer blending range parameters for blending soft objects were also proposed in [19, 11].

The above review tells that for a better shape control of the blending surface, a blending operation should possess the following properties:

(1). Provide blending range parameters to adjust the size of the resulting blending surface without deforming the overall shapes of the blended primitives;

(2). Offer C^1 continuity everywhere to generate smooth sequential blending surfaces;

(3). Allow sequential blends with overlapping blending regions.

To generate this kind of blends, the displacement method in [21] and the scale method in [11] were proposed. However, the blend, denoted as $D_k(f_1,\ldots,f_k)$, developed from the displacement and the scale methods always behaves like $Max/Min(f_1,\ldots,f_k)$ after blending on non-blending regions, this leads to the following problem:

When $D_k(f_1,\ldots,f_k)$ ***is used as a new primitive in other blends such as*** $D_2(D_k(f_1,\ldots,f_k), f_{k+1})$, ***primitives*** $f_1,\ldots,f_k$ ***always have similar subsequent blending surfaces with*** f_{k+1} ***because they always have the same blending range to blend with*** f_{k+1}. ***That is,*** $D_k(f_1,\ldots,f_k)$ ***lacks individual blending range controls on their primitives' subsequent blends.***

Therefore, to solve the above problem, this paper proposes new blends that have the ability to individually and controllably adjust the blending ranges of their primitive's subsequent blending surface, without deforming their original blending surfaces. Precisely, the newly proposed blends, denoted as $B_{Tk}(f_1,...,f_k)$, provide parameters m_i, i=1,...,k. Adjusting m_i, i=1,...,k, enables that:

(1). $B_{Tk}(f_1,...,f_k)$ behaves like $Max/Min(f_1/m_1,\ldots,f_k/m_k)$ on non-blending regions after blending.

(2). The blending surface $B_{Tk}(f_1,...,f_k)=0$ keeps unchanged whatever values m_i, i=1,...,k , are set.

As a result, when $B_{Tk}(f_1,...,f_k)$ is used as a new primitive in other blend such as $B_{T2}(B_{Tk}(f_1,...,f_k), f_{k+1})$, the shape adjustment of the subsequent blending surfaces of $f_1,...,f_k$ with f_{k+1} can be controlled by:

(1). Not only the blending range, denoted as r, for $B_{Tk}(f_1,...,f_k)$ in $B_{T2}(B_{Tk}(f_1,...,f_k), f_{k+1})$,

(2). But also parameters m_i, i=1,...,k, of $B_{Tk}(f_1,...,f_k)$ because $f_1,...,f_k$ can become $f_1/m_1,\ldots,f_k/m_k$ after the blend $B_{Tk}(f_1,...,f_k)$ in $B_{T2}(B_{Tk}(f_1,...,f_k), f_{k+1})$.

That is, $f_1,...,f_k$ have different blending ranges $r*m_1,\ldots, r*m_k$ to blend with the other primitives of $B_{Tk}(f_1,...,f_k)$'s subsequent blends.

The remainder of this paper is organized as follows. Section 2 reviews the displacement method and then describes its problem. Section 3 presents the proposed blends. Section 4 demonstrates some examples. Conclusions are given in Section 5.

2. Review

In this section, some definitions are given first. Then, the displacement method in [21] is reviewed and its problem is described.

2.1. Definitions

An implicitly defined object can be defined using primitive defining functions $f_i(X): R^3 \to R$, i=1,...,k, by

$$S(f_i, 0) \equiv \{X \in R^3 | f_i(X) \le 0\},$$

whose boundary surface $\{X \in R^3 | f_i(X)=0\}$ is denoted by either $f_i=0$ or $f_i^{-1}(0)$.

Furthermore, a multiple blend on primitives $S(f_i, 0)$, i=1,…, k, can be written as

$$S(B_k \circ F_k, 0),$$

where $(B_k \circ F_k)$ stands for $B_k(f_1,\ldots,f_k): R^3 \to R$ and is called a blending operation; F_k means $(f_1,\ldots,f_k): R^3 \to R^k$; $B_k(x_1,\ldots,x_k): R^k \to R$ is called a blending operator or function to smoothly connect blended primitives $S(f_i, 0)$, i=1,…, k.

2.2. The displacement method

In the displacement method [21], given an existing union blending operation $H_k(f_1,\ldots,f_k)$ on $S(f_i, 0)$, i=1,...,k, with blending range parameters r_i, i=1,...,k, then a displacement function D_k: $R^n \to R$ for sequential union blends can be given by

$$D_k(f_1,\ldots,f_k)=h_p \qquad \textbf{(1)}$$

where h_p is the root h of the equation

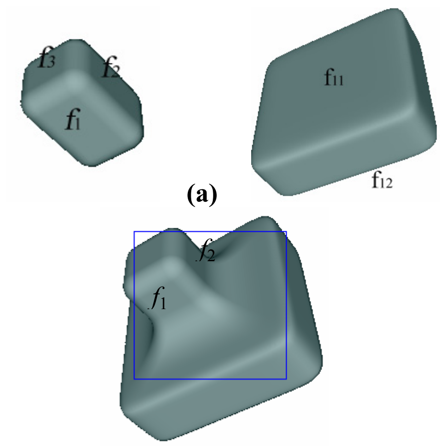
$$T(h)=H_k(f_1-h,\ldots,f_k-h)=0.$$

Because every level surface $D_k(f_1,\ldots,f_k)=h$ can be viewed as the union blending surface on $S(f_i-h, 0)$, i=1,…, k, via the blending function $H_k(x_1,\ldots,x_k)$, then $D_k(f_1,\ldots, f_k)$ can be used as a new primitive in sequential blends.

However, because $D_k(f_1,\ldots, f_k)$ in Eq. **(1)** behaves like $Min(f_1,...,f_k)$ after blending on non-blending regions, it faces the problem that

$D_k(f_1,\ldots,f_k)$ can not individually control its primitives' subsequent blending surfaces, without deforming its original shape $D_k(f_1,\ldots,f_k)=0$. That is, $D_k(f_1,\ldots,f_k)$ cannot individually control the blending ranges of their primitives' subsequent blends.

For example, as shown in Figure 1(b), primitives f_1 and f_2 of $-D_3(-f_1,-f_2,-f_3)$ in the left object of Figure 1(a) always have similar subsequent union blending surfaces with the super-ellipsoid.



(b)

Figure 1. (a) Left: an intersection $-D_3(-f_1,-f_2, -f_3)=0$ on 3 pairs of parallel planes; Right: a super-ellipsoid. (b) The union of the two objects in Figure 1(a), where f_1 and f_2 always have similar subsequent blending surfaces with the super-ellipsoid.

Similarly, the blends developed from the scale method in [11] have the same problem, too.

3. Blending Operations with Blending Range Controls on Their Primitives' Subsequent Blends

Section 3.1 presents a generalized method to develop new blending operators that can solve the problem in Section 2.2. Based the proposed method, two different blending operators are proposed in Section 3.2. Section 3.3 presents two families of Boolean set operators using the proposed operators in Section 3.2.

3.1. A generalized method

To solve the problem of the displacement and the scale methods in Subsection 2.2, a union blending operator $B_{Tk}(x_1,...,x_k)$ is required to satisfy the following requirements:

(1). $B_{Tk}(x_1,...,x_k)=0$ provides blending range parameters r_i, $i=1,...,k$, to limit the blending surface $B_{Tk}(f_1,...,f_k)=0$ in a specific blending region.

(2). $B_{Tk}(x_1,...,x_k)$ offers parameters m_i, $i=1,...,k$, to behave like $Min(x_1/m_1,...,x_k/m_k)$ everywhere on non-blending regions after blending. Since f_i becomes f_i/m_i on non-blending regions after the blend $(B_{Tk} \circ F_k)=0$, then varying m_i, $i=1,...,k$ can adjust the blending surfaces of f_i, $i=1,..,k,$ with the other primitives of $(B_{Tk} \circ F_k)$'s subsequent blends.

(3). The shape $B_{Tk}(x_1,...,x_k)=0$ always remains unchanged whatever positive values m_i, $i=1,...,k,$ are set. Thus, the shape $(B_{Tk} \circ F_k)=0$ can remain unchanged as m_i, $i=1,...,k$, are varied to adjust the blending surfaces of f_i, $i=1,..,k,$ in $(B_{Tk} \circ F_k)$'s subsequent blends.

To develop a union blending operator $B_{Tk}(x_1,...,x_k)$ that can fulfill the above requirements, a generalized method is proposed and it includes the following two steps as follows:

Step (1): Choose a k-dimensional union blending operator $H_{Tk}(x_1,...,x_k)=0$ with blending range parameters r_i, $i=1,...,k$. Precisely, its shape must be like the shape $Min(x_1,...,x_k)=0$ on non-blending regions. For example, the shape of $H_{T2}(x_1, x_2)=0$ must be like the shape shown in Figure 2.

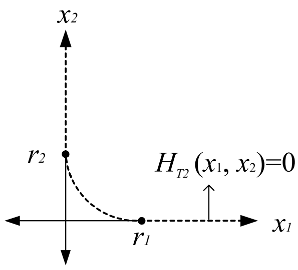


Figure 2. The shape of $H_{T2}(x_1, x_2)=0$.

Step (2): $B_{Tk}(x_1,...,x_k)$ can be given by

$$B_{Tk}(x_1,...,x_k)=h_p \qquad (2)$$

where h_p is the root h of the equation

$$T(h)=H_{Tk}(x_1-m_1h,...,x_k-m_k-h)=0. \qquad (3)$$

In fact, every level surface $B_{Tk}(x_1,...,x_k)=h$, $h\in R$, is similar to the surface $H_{Tk}(x_1,...,x_k)=0$ translated by $M=[m_1,\ldots,m_k]$. For example, using the curve in Figure 2 as $H_{T2}(x_1, x_2)=0$, level curves $B_{T2}(x_1,x_2)=h$, $h\in R$, in Eq. (3) can be shown as the dotted curves in Figure 3.

Some properties of $B_{Tk}(x_1,...,x_k)$ from Eq. (2) are presented as follows:

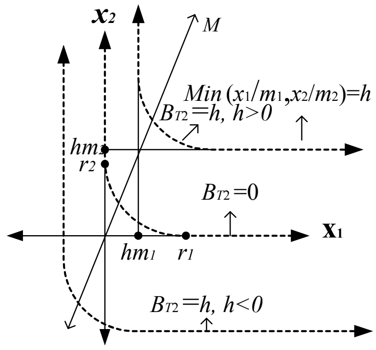


Figure 3. Level curves $B_{T2}(x_1, x_2)=h$, $h\in R$, where the curve in Figure 2 is used as $H_{T2}(x_1, x_2)=0$.

(1). In non-blending regions, $B_{Tk}(x_1,...,x_k)$ behaves like $Min(x_1/m_1,...,x_k/m_k)$, and hence a primitive f_i becomes f_i/m_i after the blend $(B_{Tk} \circ F_k)=0$. This is because $H_{Tk}(x_1,...,x_k)=0$ is $Min(x_1,...,x_k)=0$ on non-blending regions. Then, solving the root h of the equation $T(h)=Min(x_1-m_1h,...,x_k-m_kh)=0$ yields the function $Min(x_1/m_1,...,x_k/m_k)$.

(2). Setting zero to h of the equation $T(h)=H_{Tk}(x_1-m_1h,...,x_k-m_kh)=0$ in Eq. (3) yields that the shape $B_{Tk}(x_1,...,x_k)=0$ is always the same as $H_{Tk}(x_1,...,x_k)=0$, whatever $[m_1,\ldots,m_k]$ is set.

(3). In blending regions, all surfaces $B_{Tk}(x_1,...,x_k)=h$, $h\in R$, always have the same arc-shaped surface with constant blending ranges $r_1,\ldots,$ and r_k, the same as those of $H_{Tk}(x_1,...,x_k)=0$. The reason is because $B_{Tk}^{-1}(h)$ is similar to $H_{Tk}(x_1,...,x_k)=0$ translated by $[m_1,\ldots,m_k]$.

All these above indicate that $B_{Tk}(x_1,...,x_k)$ from Eq. (2) can solve the problem of the displacement and the scale methods stated in Subsection 2.2.

3.2. Real cases of the blending operators with blending range controls on their primitives' subsequent blends

Section 3.1 only gives a generalized method to generate blending operators with blending range controls on their primitives' subsequent blends. Based on that method, this section presents some real operators.

3.2.1. Two-dimensional blending operators. When $H_{T2}(x_1, x_2)$ in **Step (1)** is given by

$$\begin{cases} H_H(x_1,x_2)=0 & \text{region II} \\ Min(x_1,x_2)=0 & \text{otherwise} \end{cases}$$

where $H_H(x_1,x_2)= r_2^2x_1^2+r_1^2x_2^2+r_2^2r_1^2-2r_1r_2^2x_1-2r_1^2r_2x_2+2px_1x_2$, $-\infty<p< r_1r_2$, and region **II** is $\{(x_1, x_2)\in R^2 \mid m_2x_1-m_1(x_2-r_2)>0$ and $m_2(x_1-r_1)-m_1x_2<0\}$,

then in Eq. (2), a 2D $B_{T2}(x_1, x_2)$ can be given by solving the quadratic formula of $H_H(x_1-m_1h,x_2-m_2h)=0$ and written as

$$\begin{cases} (-b \pm (b^2 - 4ac)^{0.5})/(2a) & \text{if } a \neq 0 \text{ and on region II} \\ -c/b & \text{if } a = 0 \text{ and on region II} \\ Min(x_1/m_1, x_2/m_2) & \text{otherwise} \end{cases} \quad \textbf{(4)}$$

where $a=r_1^2m_1^2+r_2^2m_2^2+2pm_1m_2$;

$b=2((r_2-x_2)r_1^2m_2+(r_1-x_1)r_2^2m_1-(x_1m_2+x_2m_1)p)$;

$c=(r_1x_2+r_2x_1-r_1r_2)^2+2(p-r_1r_2)\, x_1x_2$.

3.2.3. High-dimensional blending operators. When $H_{Tk}(x_1,\ldots,x_k)$ in **Step (1)** is given by

$$H_{Tk}(x_1,\ldots,x_k)=\sum_{i=1}^{k}[(r_i - x_i)/r_i]_+^{p_i} - 1 = 0,$$

where $[*]_+ \equiv Max(0,*)$ and both $r_i>0$ and $p_i>1$ hold for $i=1,\ldots,k$,

then in Eq. (2), a hyper-ellipsoidal union operator $B_{Tk}:R^k \rightarrow R$ with blending range parameters $r_1,\ldots,r_k$ and curvature parameters $p_1,\ldots,p_k$ for simultaneous multiple blends can be given by

$$B_{Tk}(x_1, \ldots, x_k)= h_p, \ \ h_p \in T^{-1}(0) \qquad \textbf{(5)}$$

where

$$T(h)=H_{Tk}(x_1-m_1h,\ldots,x_k-m_kh) = \sum_{i=1}^{k}[(r_i - x_i + m_ih)/r_i]_+^{p_i} - 1,$$

and $m_i>0$ for $i=1,\ldots,k$.

To solve the root h_p of the equation $T(h)=0$ in Eq. **(5)**, one can apply the Regula Falsi method and use $h \in (Min_{i=1}^{k}((x_i-r_i)/m_i), Min_{i=1}^{k}(x_i/m_i)]$ as the initial guesses. $Min_{i=1}^{k}(*_i)$ means $Min(*_1,\ldots,*_k)$ for short.

3.3. Dual forms

Eqs. (4) and (5) in Section 3.2 offer union blending operators only. To develop a full family of Boolean set operators, one can adopt the dual forms of Eqs. (4) and (5), which is written by:

(1). Union operation: $B_{Tk}(f_1, \ldots, f_k)$.
(2). Intersection operation: $-B_{Tk}(-f_1, \ldots, -f_k)$.
(3). Difference of $S(f_1, 0)$ from $S(f_2, , 0),\ldots, S(f_k, 0)$: $-B_{Tk}(-f_1, f_2,\ldots, f_k)$.

4. Demonstration

Some examples about the blending range control on primitives' subsequent blends of the dual forms of Eqs. (4) and (5) are presented in this section.

Example 1: Varying $[m_1,\ldots,m_k]$ of B_{Tk} in Eqs. (4) and (5) can individually adjust the blending ranges of the primitives $f_1, \ldots, f_k$ of the intersection operation $-(B_{Tk} \circ -(F_k))=0$ in a union blend. This can be seen in Figure 4, where m_2 for f_2 of B_{Tk} is decreased from 1.2, 0.9, 0.6, to 0.3 to change the subsequent union blending surface of $f_2=0$ with the super-ellipsoid for the objects from top left to bottom right.

Example 2: Varying $[m_1,\ldots,m_k]$ of B_{Tk} in Eqs. (4) and (5) can individually adjust the blending ranges of the primitives $f_1, \ldots, f_k$ of the intersection operation $-(B_{Tk} \circ -(F_k))=0$ in a difference blend. This can be seen in Figure 5, where m_2 for f_2 of B_{Tk} is decreased from 1.2, 0.9, 0.6, to 0.3 to change the subsequent difference blending surface of the super-ellipsoid from $f_2=0$ for the objects from top left to bottom right.

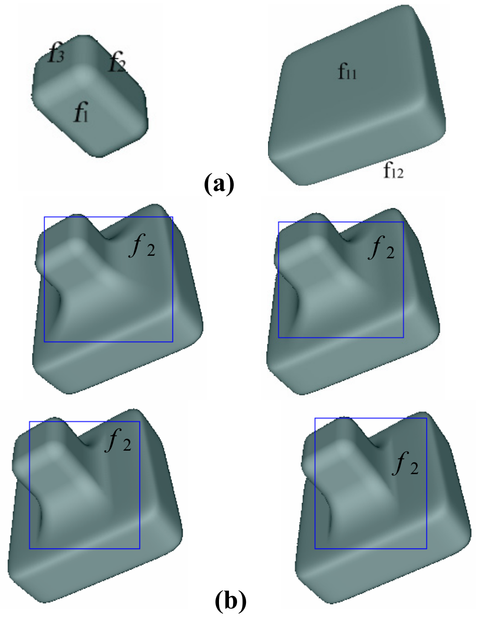


Figure 4. (a) Left: An intersection $-(B_{T3} \circ -(F_3))=0$ on 3 pairs of parallel planes; Right: A super-ellipsoid. (b) The shape changes of the union of the two objects in Figure 4(a), where only the subsequent blending surface of f_2 with the super-ellipsoid, in the marked regions, are shrunk gradually, but the shape $-(B_{T3} \circ -(F_3))=0$ remains unchanged as m_2 for f_2 is decreased from 1.2, 0.9, 0.6, to 0.3 for the objects from top left to bottom right.

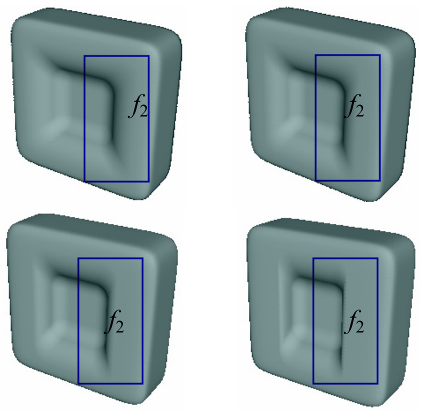
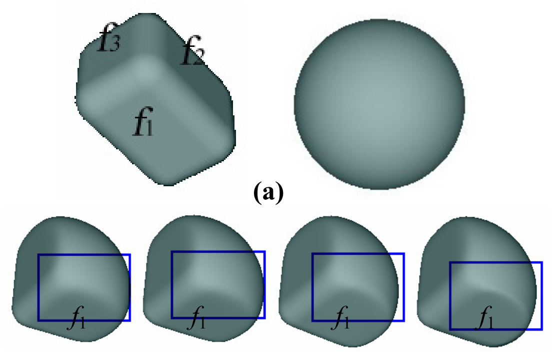


Figure 5. The shape changes of the difference of the super-ellipsoid from the $-(B_{T3} \circ -(F_3))=0$ in Figure 4(a), where only the edges, cut by $f_2=0$, in the marked regions, become sharper gradually, as m_2 for f_2 is decreased from 1.2, 0.9, 0.6, to 0.3 for the objects from top left to bottom right.

Example 3: Varying $[m_1,\ldots,m_k]$ of B_{Tk} in Eqs. (4) and (5) can individually adjust the blending ranges of the primitives f_1, …, f_k of the intersection operation - $(B_{Tk} \circ -(F_k))=0$ in an intersection blend. This can be seen in Figure 6, where m_1 for f_1 of B_{Tk} is decreased from 1.2, 0.8, 0.45 to 0.15 to change the subsequent intersection blending surface of $f_1=0$ with the ball for the objects from left to right.



(b)

Figure 6. (a) Left: An intersection -($B_{T3} \circ$ -(F_3))=0 on 3 pairs of parallel planes; Right: A ball. (b) The shape changes of the intersection of the two objects in Figure 6(a), where only the edges, in the marked regions, of f_1 intersecting with the ball become sharper gradually, but the shape -($B_{T3} \circ$ -(F_3))=0 remains unchanged as m_1 for f_1 is decreased from 1.2, 0.8, 0.45, to 0.15 for the objects from left to right.

5. Conclusions

In implicit surface modeling, most of the existing blends remain their primitives' properties unchanged on non-blending regions after blending. This causes that when they are used as a new primitive in other blends, the shape controls on their primitives' subsequent blending surfaces rely solely on the subsequent blending operators, and hence some difficulties in the blending range controls on their primitives' sequential blends develop, as stated in Subsection 2.2. To solve this problem, this paper has proposed some new blends that can provide parameters to adjust their primitives' subsequent blending surfaces, respectively, without deforming their original shapes, when they are reused as a new primitive in other blends. On the contrary, other existing blends do not have this kind of ability.

Furthermore, to develop this blend stated above, a generalized method has been proposed in this paper. The proposed method can transform some existing union blending operators into a new blending operator that has the following abilities:

(1). Provide blending range parameters to adjust the size of the resulting blending surface, without deforming the overall shapes of the blended primitives;

(2). Offer C^1 continuity over the entire domain to generate smooth blending surfaces;

(3). Allow sequential blends with overlapping blending regions.

(4). Provide parameters to adjust their primitives' subsequent blending surfaces respectively, without deforming its original blending surface.

Based on the proposed method, one two-dimensional operator (conics) and one high-dimensional blending operator (super-ellipsoids) have been developed in this paper. These two kinds of operators give more parameters to adjust the resulting blending surface than the other existing operators do. Especially, the proposed operators also provide curvature parameters to adjust the shapes of the transitions of the blending surfaces.

In fact, in Section 3.1, the generalized method does not always generate a differentiable function. It depends on the chosen union operator in **Step (1)**. Therefore, future works should focus on finding a rule for choosing a union operator such that the resulting blending operator (function) in **Step (2)** can be assured to be differentiable everywhere.

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