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A Method of Construction Invariant Features for AI Image Analysis Systems

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Abstract

The invariant recognition of shapes under affine distortion and translation requires the use of invariant descriptors under these distortions. This paper is concerned with the definition of classes of such features which are appropriate for the description of 2D shapes and could be used for problems like character recognition, even for cases where the characters are very complex like Chinese characters, for example. As we are concerned with the problem of classes of features as opposed to individual features themselves, we are in position to propose very large numbers of features that belong to these classes. These features do not necessarily have a straight forward geometric interpretation as they are derived algebraically. Therefore it is difficult to investigate their usefulness theoretically, so they have to be analyzed by AI systems before being used for Pattern Recognition.

1 Introduction

This paper concerns the problem of finding invariants or quantities that are invariant under planar affine transformations of images. These invariants are to be used in complex character recognition.

Every function defined on a set of images can be called a *feature*. Therefore the feature characterizes images by its own way. Using this term, any kind of information about an image can be called by feature. Then we restrict ourselves to *integral features*. This term cannot be absolutely definite, but usually it is understood that this is a feature which can be computed using simple mathematical formulas without many logical operations. For example, statistical characteristics of images are integral features. To have a better idea about integral features, let us recall that they can be easily computed by massively parallel algorithms. Integral features are indifferent to color restrictions, good boundary and noise. Therefore, they seem to be prominent.

There were many papers devoted to integral features at the beginning of Pattern Recognition theory; we can read about this in old surveys. For example, such features as square, length of boundary, projections, center of gravity, characteristics of color histogram, moments, fractal dimensions, mean curvature of the boundary, middle distance between black points, coordinates of center of gravity – they are integral features. Unfortunately, it is well-known, that these approaches had no considerable success. Nowadays investigators prefer to examine structure of images more carefully, using more complicated ideas than simple math.

The point of our studying began when we discovered a simple mathematical form of presenting all known integral features. This form involves three functionals into one formula. Then this formula, consisting of a *triplet of functionals*, provides us with *triple feature*. So every triple feature is a function, defined on images, and this function has a special algebraic form (see below).

The first result of this form of triple feature is the following. Suppose we have 100 samples of functional; they can be found easily. Combining them by three into triplets, we immediately receive $100\times100\times100$ = one *million* of triple features. We see that the situation is sufficiently different from previous papers on integral features. Authors explored there integral features and every time they observed a *few* number of features, see [1 – 3]. Apparently, they had an advantage of knowing the meaning of the features under their consideration. However, it is impossible to know the meaning of the million features.

Having many features, we hope sooner or later to resolve a given pattern recognition task. We perform our experiments in this paper to demonstrate this. The first experiment is concerned with a hieroglyph recognition problem and finding out positions, rotations and scaling of distorted patterns. The second one deals with classes of human blood cells. In the second experiment the problem was not to recognize (like we have done before with hieroglyphs), but to determine classes. We prepared 5 classes of blood cells by lists of patterns, and sometimes it was not easy to find out the class of every new cell with the naked eye. However, 120 of the triple features did recognize the class of every new cell. We do not know how they did it, because we do not know the meanings of triple features.

One expects that some triple features are not good for a given recognition task, but there can be triple features which fit well. And this can easily be seen from our experiments. So the problem of sorting and investigating of triple features arises. This is a reason why we present our paper to this conference on artificial intelligence (AI). We resolved the presented simple tasks taking many triple features without any analysis; we used good features and bad ones as well. We see that this works; however, it is obvious that this method can be used for real tasks if only a special system for analysis of triple features would be made by specialists on AI.

2 Attributes of functionals

In the base of our theory, there is a simple mathematical construction; therefore, we can soon explain it, leaving for a while more complicated results and strict definitions. To construct a good triple feature we should know something about the triple of functional involved. Functionals can have properties we are interested in. The problem of constructing new triple features is charged upon a computer; therefore, formulas of functionals should be presented to a computer's program, and their properties also should be described in the program. We call these properties we are interested in by "attributes" of functionals. Below we give strict definitions, now we give an example. A functional is a map, defined on a set of usual functions of one variable, say x. Let Ξ be a functional; for example, let it use the formula $\Xi(\xi(x))= \left(\int|\xi(x)|^p dx\right)^q$ for some fixed p and q. We see from this that $\Xi(\xi(ax)) = a^{-q}\ \Xi(\xi(x))$ and $\Xi(c\xi(x)) = c^{pq}\ \Xi(\xi(x))$ for all positive a and c. These exponents $-q$ and pq are attributes of the functional Ξ and they are designated $\kappa=\kappa(\Xi)= -q$ and $\lambda=\lambda(\Xi)=pq$. We refer to these numbers as *exponents of functionals.* We call $\kappa(\Xi)$ to be *penetrating homogeneity* exponent and $\lambda(\Xi)$ *homogeneity* exponent of the functional Ξ. An arbitrary functional may have not one of these two exponents or have not both. This information goes to the computer.

There are very common properties of functionals: ($i1$) $\Xi(\xi(x+b))= \Xi(\xi)$ (for all permissible functions and numbers b) and ($s1$) $\Xi(\xi(x-b))= b+\Xi(\xi)$ (for all ξ and b). Our example $\Xi(\xi(x))= \left(\int|\xi(x)|^p dx\right)^q$ fits to the first. In our terminology, "the functional Ξ is invariant (to shifts of underlying functions along the axis of independent variable)."

These attributes will be used in the first result – Theorem 1. Later we introduce more attributes. Mathematically, our theory consists in introducing more attributes of functionals and inventing theorems like Theorem 1 (see below). Functionals that we use must reveal some features of functions of one variable; in other words, they are intended to be used for telling one function from another. One sees that in this paper all of the used functionals have attributes. The following question arises. Surely we can invent good functionals to detect features of functions. But will these good functionals have good attributes? Will these functionals go to the theory of triple features? Our answer is the following. If we have a functional Ξ without some attributes, and we want to have these attributes, we can easily remake this functional Ξ into a new one Ξ^* which will have the desired attributes and will have similar reactions to underlying functions. This way is called "normalizing" of functionals. We do not explicitly use this way in this paper; however, let us give a notion of it with a plain example. Suppose we want a functional Ξ^* having a property $\lambda(\Xi^*)=0$. For this we can normalize Ξ in ways $\Xi^*(\xi(x))= \Xi(\xi(x)/max\xi)$ or $\Xi^*(\xi(x))= \Xi(\xi(x)/\int\xi(x)dx)$. It is easy to check that we have $\lambda(\Xi^*)=0$. There is no a standard way, but there are many methods, depending on circumstances. This shows that we have a sufficient source of functionals for constructing triple features; here we are not restricted by the mathematical nature of functionals.

3 Structure of triple features

The idea of triple features is based upon a well-known parametrization of lines in the plane, so we have to touch this known theme. Let F be an image function defined on the plane. Every point M of the plane has rectangular coordinates (x,y) and polar coordinates r and φ connected by formulas $x= r\cos\varphi$, $y= r\sin\varphi$. The number $F(x,y)$ is the color or brightness in the point having coordinates (x,y) of the image described by its image function F. Below we use the standard parametrization of a line $l=l(p,\varphi)$ in the plane in the form

parametrization of the line $l(p,\varphi)$ with parameter t:

$$L(t,p,\varphi)=(p\cos\varphi-t\sin\varphi,\ p\sin\varphi+t\cos\varphi), \qquad (1)$$

where t is a natural parameter on the line. The distance between the origin having coordinate (0,0) and the line l is $|p|$. The point of the line l with coordinates ($p\cos\varphi$, $p\sin\varphi$) is the nearest to the origin with coordinates (0,0). We can consider $L(t,p,\varphi)$ being a vector-function of three independent variables. This function is a parametrization of the line l which we call the *line with polar attributes p and* φ. This line has a direction defined by the direction of movement of the point $L(t,p,\varphi)$ when the parameter t increases. This direction also can be defined with the unit vector having coordinates $-\sin\varphi$ and $\cos\varphi$. Therefore we consider two lines $l(p,\varphi)$ and $l(-p,\varphi+\pi)$ being different because of their different directions in spite of they coincide geometrically.

Now we can introduce triple features. To make a formula for a triple feature we must have three functionals prepared. Designate them by **T**, **P** and **Φ**. We call them *trace functional* **T**, *diametrical functional* **P** and *circus functional* **Φ**. The first two of them, **T** and **P**, should be defined on two sets of functions, and every function f of these two sets is finite supported (i.e. it has to be equal to zero for all arguments remote from the origin 0 of the real line **R**). The third functional is defined on a set of 2π- periodic functions of the real line **R**. To simplify our notation, suppose that these functionals are applied to only those functions whose independent variables are designated by t, p and φ respectively. That is notations like $\mathbf{T}f$, $\mathbf{P}g$ and Φh stand for $\mathbf{T}(f(t))$, $\mathbf{P}(g(p))$ and $\Phi(h(\varphi))$ respectively. Also, it means that if functions f, g and h have more variables, designated by others letters, then these additional variables are considered to be fixed while one of the functionals is applied, because these functionals can be applied to functions of one variable only.

Mathematically the *triple feature* (designate it by Π), made out of the triplet of functionals (**Φ**, **P**, **T**) for the image function F, has a simple form of

$$\Pi(F)= \Phi\circ\mathbf{P}\circ\mathbf{T}\circ F\circ L(t,p,\varphi), \qquad (2)$$

where the sign "o" means map's composition. To get a the number $\Pi(F)$, we involved the functionals. Therefore we can use also notation $\Pi(F)= \Pi(F, \Phi, \mathbf{P}, \mathbf{T})$.

Let us explain formula (2). Formula (2) begins to work at the composing a function $f(t,p,\varphi)= F(L(t,p,\varphi))$ of three variables. This is the color (or brightness) of the image in the point having coordinates (1). Next step is receiving the function $g(p,\varphi)= \mathbf{T}f(t,p,\varphi))$ of two variables; notice, please, that the functional **T** is applied to $f(t,p,\varphi)$ while p and φ are fixed; that is, in computers the functional **T** must be applied many times, to exactly how many pairs (p,φ) are involved. We discuss this below. Then we apply the diametrical functional and get a 2π-periodic function $h(\varphi)= \mathbf{P}g(p,\varphi)$. At the last step we have the triple feature $\Pi(F)= \Phi h(\varphi)$.

4 Computation of triple features

Consider an image set in a circle with radius r. For points situated out of the circle let the image function F be equal to zero. We take a finite set of lines $l(p_k,\varphi_j)$, (k=1..K, j=1..J) so that we have equidistant grids $-r=p_1< p_2<...< p_K=r$ and $0=\varphi_1< \varphi_2<...< \varphi_J<\varphi_{J+1}=2\pi$. Every line $l(p_k,\varphi_j)$ has points $L(t, p_k,\varphi_j)$ (see (1)) on it, where t ranges in a grid of a segment $[-r,r]$. Therefore for this line $l(p_k,\varphi_j)$ we have a finite set of values $f(t, p_k,\varphi_j)$ for a finite grid of parameter t. This allows us to compute the trace functional **T** for this function f, then we get numbers $g(p_k,\varphi_j)$. These numbers make a *trace matrix*; it has J columns and K lines. Then every column, representing values of the function $g(p,\varphi_j)$ for finite set of parameter p, goes under the diametrical functional **P**. Therefore we get $h(\varphi)$ for $\varphi= \varphi_1, \varphi_2,..., \varphi_J$. At last, we can compute $\Pi(F)$ using the circus functional **Φ** at $h(\varphi)$.

5 Invariant triple features

Theorem 1. (*a*) Let **T**, **P** and **Φ** be three functionals described in section 3. Let the three are invariant in sense (*i*1) (see section 2). Then the triple feature $\Pi(F, \Phi, \mathbf{P}, \mathbf{T})$ (see (2)) is independent of any rotation and shift of image with image function F.

(*b*) Let two images be given having image functions F_1 and F_2. Suppose the second image is made out of the first image with rotation, sizing with a positive coefficient μ (if $\mu<1$, then the second new image is smaller than the first image) and shifting to a vector. If **T**, **P** and **Φ** are invariant in the sense of (*i*1), and there exist four attributes $\kappa(\mathbf{T})$, $\kappa(\mathbf{P})$, $\lambda(\mathbf{P})$ and $\lambda(\Phi)$, then the triple features (2) for the two images are connected with the following relation

$$\Pi(F_2)= \mu^{\omega}\Pi(F_1), \textit{ where } \omega=(\kappa(\mathbf{T})\lambda(\mathbf{P})+\kappa(\mathbf{P}))\lambda(\Phi). \qquad (3)$$

Theorem 1 (*b*) gives us a plan for pattern recognition. Take many triple features having $\omega=0$, they react only to shape neglecting size and shift of images. Therefore these features characterize images by vectors (every component of which is one of the triple features) and these vectors are the same for patterns, for sized patterns and shifted patterns. Therefore we can recognize patterns like ones presented on Fig 1.

After a successful recognition process we can use a set of triple features having $\omega\neq0$. Using Theorem 1 (*b*) we can find out the sizing parameter μ. We do this in the first experiment concerning Fig.1.

Theorem 1 (a) covers all results of stereometrical metallography and stochastic geometry [1–3] in their application to image analysis. Their method is based on mean characteristics of intersection of an arbitrary line and an image. Designate this characteristic by **T** and then we see that theory of [1–3] is the theory of triple features (2) where we assumed $\mathrm{P}g=\int g(p)dp$ and $\Phi h(\varphi)=\int_{[0,2\pi]}h(\varphi)d\varphi$. By choosing several different variants of **T** they got several geometrical characteristics of images. However, having **P** and **Φ** fixed, their method is not able to use the advantage of combinatorial multiplication as it holds for triple features.

6 More theorems

In previous sections we explained the main ideas of our method and below we give a more formal and brief account.

Now let us discuss how images can change their brightness while they are being distorted. Let F_1 be a function of a toned image, that is the F_1 is a function defined in the plane which gives the whole information about the image. Let the F_1 describes the brightness of the image or its color. We refer to the image with the same designation F_1. Consider another image F_2, which is made of the initial image F_1, by sizing with coefficient μ. Then let a movement be applied. That is, every point of the initial image is translated with a fixed beforehand linear operator and a shift. The whole transformation is an *affine conformal transformation Aff* of the initial image F_1. For example, if the distance between the camera and the image is increased in two times, we have $\mu=0.5$. Let M be a moving point in the plane. It is easy to see, that we can write $F_2(M)= F_1(Aff^{-1}M)$.

However, distortion of images can provide changing in their brightness. We may have another formula $F_2(M)= \mu^{\beta}F_1(Aff^{-1}M)$, where β is a *brightness exponent*. This exponent depends on a case we consider. For example, the light in the image is unchanged and F_2 is got while changing the distance. We see that $\beta=0$ here. Now suppose that the light is attached to the camera. A simple computation shows that $\beta=2$. Another example is if the image F is painted on some ribbon film. If it is shrunk while the distance of the view is constant, we receive the small and more bright image F_2 because the paint is made dense. In the latter case we have $\beta=-2$.

We conclude the parameter β can be constant for a recognition task and it is known beforehand. Assuming the *hypothesis of brightness exponent* we take the parameter β being fixed. In [4, 5] there was assumed $\beta=0$. This case is called the *hypothesis of color independence for linear distorted images* [6]. Now assume the hypothesis and fix the brightness exponent β.

Now we give several definitions. A functional Ξ is called *invariant* if

(i1) $\Xi(\xi(x+b)) = \Xi\xi$ for all real b. It may have additional properties:

(i2) There exists a positive function α and $\Xi(\xi(ax)) = \alpha(a)\Xi\xi$ for all $a>0$ and $a\in\mathrm{Dom}(\alpha)$; in any case $1\in\mathrm{Dom}(\alpha)$;

(i3) There exists a positive function γ and $\Xi(c\xi)= \gamma(c)\Xi\xi$ for all $c>0$ and $c\in\mathrm{Dom}(\gamma)$; in any case $1\in\mathrm{Dom}(\gamma)$.

It is always possible a case $\mathrm{Dom}(\alpha)=\{1\}$ or $\mathrm{Dom}(\gamma)=\{1\}$. Another natural case is if the domain is $(0,\infty)$. It is easy to prove that, assuming the functions α and γ are continuous at least in one point, we can find numbers $\kappa=\kappa(\Xi)$ and $\lambda=\lambda(\Xi)$ such that (see (i1) and (i2)) $\alpha(a)= a^{\kappa(\Xi)}$ and $\gamma(c)= c^{\lambda(\Xi)}$. We refer to these numbers as *exponents of invariant operator*. Functions α and γ are *penetrating homogeneity function* and *homogeneity function* respectively. The same terminology is held for the exponents.

We call a functional Z *sensitive* if

(s1) $Z(\varsigma(x+b))= Z\varsigma-b$ for all real b.

It may have additional properties:

(s2) $Z(\varsigma(ax))= (1/a)\, Z\varsigma$ for all positive a; combining this and the (s1), we obtain $Z(\varsigma(a(x+b)))= (1/a)Z\varsigma-b$ or $Z(\varsigma(ax+b))= (1/a)(Z\varsigma-b)$.

(s3) $Z(c\varsigma)=Z\varsigma$ for all positive c.

We call a functional Z τ-*sensitive* if

(s1.τ) $Z(\varsigma(x+b))= Z\varsigma-b$ (modd τ) for all real b.

We can combine (s1.τ) and (s3) but (s2). A τ-sensitive functional is applied to 2π-periodic functions in this paper.

Now we give results of [4], generalized with the hypothesis of brightness exponent. Let Π_1 be a triple feature for the initial image and Π_2 be the same triple feature for a conformal distorted image made out of the initial one with a homothety, a rotation, and a shift. The results are shown in Table 1.

Let us give explanations for this table The second image is stretched in μ times (if $\mu>1$; or shrunk, if $0<\mu<1$) comparatively with the initial image and rotated to an angle θ. The number n in the Table can be any natural number, $\nu=1/\mu$. The operation $u^{\perp}$ is elimination of the first Fourier harmonic from the underlying periodic function u.

7 Finding shift parameters

It is known that center of gravity simplifies recognition of simple objects. Problem is that sometimes we cannot find it properly because images can be damaged. One of advantage of our theory is that we can avoid using centroid coordinates. Therefore we do not use center of gravity. In this section we show that we can find displacements of images using triple features.

Table 1 Theorems

Property of T	Property of P	Property of Φ	Connection Π_2 and Π_1
(i1); $\alpha_T(\nu)$ and $\gamma_T(\mu^\beta)$ can be computed	(i1); $\alpha_P(\nu)$ and $\gamma_P(\gamma_T(\mu^\beta)\alpha_T(\nu))$ can be computed	(i1)	$\Pi_2=\gamma_\Phi(\gamma_P(\gamma_T(\mu^\beta)\alpha_T(\nu))\alpha_P(\nu))\Pi_1$ (if the coefficient can be computed)
see above	see above	($s1.2\pi/n$); (if $\gamma_P(\gamma_T(\mu^\beta)\alpha_T(\nu))\alpha_P(\nu)\neq 1$ then (s3))	$\Pi_2=\Pi_1+\theta$ (modd $2\pi/n$)
see above	(s1); (if $\mu\neq 1$ then (s2)); (if $\gamma_T(\mu^\beta)\alpha_T(\nu)\neq 1$ then (s3))	(i1); $\Phi u=\Phi u^\perp$ for all permissible u	$\Pi_2=\gamma_\Phi(\mu)\Pi_1$ (if the coefficient can be computed)
see above	see above	($s1.2\pi/n$); (if $\mu\neq 1$ then (s3)); $\Phi u=\Phi u^\perp$ for all permissible u	$\Pi_2=\Pi_1+\theta$ (modd $2\pi/n$)

Let an initial image function F_1 be given and a distorted image function F_2 be given. Suppose that the distorted image was received by rotating to an angle θ, sizing with a coefficient μ and then shifting the initial image. Let the shift vector have coordinates $(x, y)^t=(s_0\cos\psi_0, s_0\sin\psi_0)^t$. Let **T** satisfy ($i$1) and ($i$2). Let **P** satisfy ($s$1), ($s$2) and ($s$3). After computation **T** and **P** in (2) we receive a 2π-periodic function $h(F, \varphi)$. We call it *circus* for F. Under the pointed out conditions we can derive the following relation of circuses: $\mu h(F_1,\varphi-\theta)+s_0\cos(\varphi-\psi_0)=h(F_2,\varphi)$. After the recognizing procedure explained above, and finding out parameters μ and θ according Table 1, we are able to find displacement parameters s_0 and ψ_0 using this formula. Let C_a and C_b be circus functionals giving coefficients a_1 and b_1 of the first Fourier harmonic of a 2π-periodic function. Define two triple features $\Pi_a(F)=C_a(\mathbf{P}(\mathbf{T}(F(L(\varphi,p,t)))))$ and $\Pi_b(F)=C_b(\mathbf{P}(\mathbf{T}(F(L(\varphi,p,t)))))$. Taking only first harmonics, we rewrite our the relation in the form $\mu\Pi_a(F_1)\cos(\varphi-\theta)+\mu\Pi_b(F_1)\sin(\varphi-\theta)+s_0\cos(\varphi-\psi_0)=\Pi_a(F_2)\cos(\varphi-\theta)+\Pi_b(F_2)\sin(\varphi-\theta)$. In this formula, we know all numbers, but s_0 and ψ_0. The parameters s_0 and ψ_0 of the shift can be immediately found from this formula.

8 Pattern recognition with triple features

Using results of Table 1 and explanations in the previous sections, one can organize the pattern recognition process. Therefore we give only a brief account of our experiments, to show the main steps, and results. Presented tables give the notion what kind of numerical information we get working with triple features.

To get a triple feature we need a triple of functionals. The first of them is a trace functional **T**. Samples of trace functionals are presented in Table 2. They all are invariant. In Table 3 there are samples of a diametrical functional **P** used in this experiment. Two samples (namely D3 and D6) are sensitive diametrical functionals, the others are invariant. These sensitive functionals are used to reveal an angle of the pattern's rotation and parameters of shifts of patterns. A circus functional Φ is applied to 2π- periodic functions; therefore it has no exponent $\kappa(\Phi)$, but it may have an exponent $\lambda(\Phi)$. The samples of it are shown in Table 4. We see there the amplitudes of Fourier harmonics and other functionals.

Samples of the patterns are denoted by p1, p2, .. p8. The distorted (twisted, sized and shifted) patterns are denoted by q1..q8 respectively. They are shown in Figure 1. The distorted images are made from p1..p8 by sizing with coefficients μ1, μ2, .., μ8 respectively. Then rotating to angles θ1, θ2, .., θ8 and translating.

Our aim is to recognize* the patterns q1..q8 and to find out sizing coefficients μi (i=1..8) and angles θi (i=1..8). Then shifts have to be found. This sequence is offered by the theory of triple features. Notice that the usual way is quite the reverse: to find a shift at first, then sizing and rotation, and at last to recognize the image.

The lists of functionals in Tables 3-4 are input into a PC with attributes listed in those tables. So a PC program is able to choose triple features having the exponent $\omega=0$ in formula (3). Columns with these triple features characterize shapes without accounting their size, rotation and shift. To proceed with the recognition procedure, we need to introduce a distance in the criteria space of these columns. We use L^1-distance. The result is presented in Table 5. The diagonal of the Table consists of the smallest numbers in each row. It means that the recognizing procedure is a success.

Table 2 Samples of Trace Functional

Trace functional $\mathbb{T}$		
No	$\mathbb{T}(u(t))$	$\kappa(\mathbb{T})$
T1	$\int u(t)dt$	-1
T2	Length of the maximal segment in supp(u)	-1
T3	mes(conv(supp(u)))	-1
T4	Standard deviation of $\lvert u(t)\rvert / \int \lvert u(t)\rvert dt$ or 0 if $u \equiv 0$	-1
T5	T1*T4	-2

Table 3 Samples of Diametrical Functional

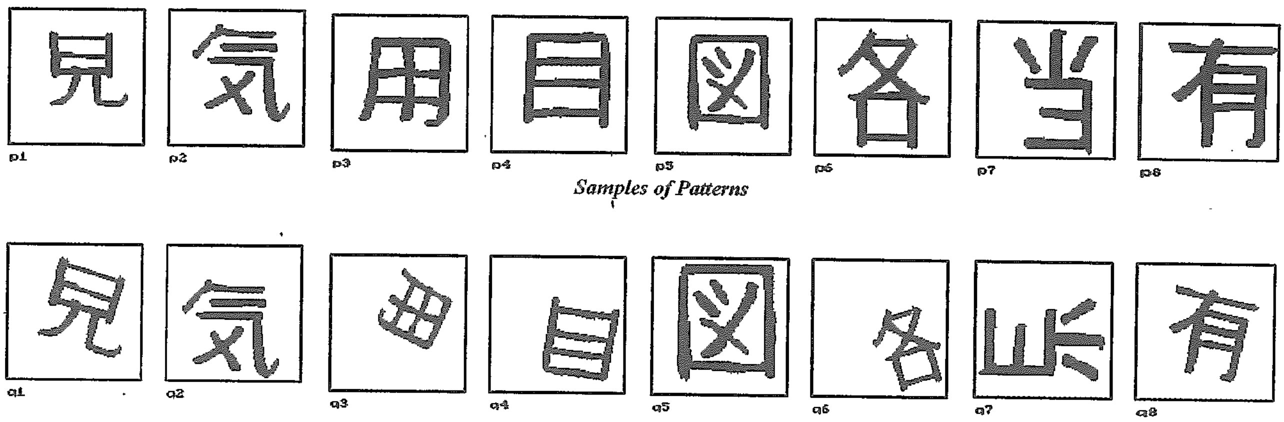
Diametrical functional $\mathbb{P}$			
No	$\mathbb{P}(u(p))$	$\kappa(\mathbb{P})$	$\lambda(\mathbb{P})$
D1	L^2 norm of u	-0.5	1
D2	Max(u)	0	1
D3	Middle point of $u/\int u(p)dp$	-1	0
D4	mes(supp(u))	-1	0
D5	Max $\lvert du/dp \rvert$	-1	1
D6	Min(Supp(u))	-1	0

Mathematically, these small numbers should be equal to zero. However, we have quantization in numbers of lines l (see above) and points in these lines. There were 40×30 lines in this experiment. However every line is taken into account two times. Therefore we deal with trace matrices consisting of 20×30 independent numbers. That is, we computed trace matrices having size of 20×30 only. It explains why the diagonal elements in Table 5 are not equal to zero. Table 5 shows that only 20×30 numbers are sufficient to recognize the shapes of our pictures. The maximal numbers in the diagonal of Table 5 are *dist*(q3,p3) and *dist*(q6,p6). This is explained by the size of the images q3 and q6 being smallest. Therefore only few lines of the 20×30 intersect them.

Table 4 Samples of Circus Functional

Circus Functional Φ					
No	$\Phi(u(\varphi))$	$\lambda(\Phi)$	No	$\Phi(u(\varphi))$	$\lambda(\Phi)$
C1	$(1/2\pi)\int u(\varphi)d\varphi$	1	C7	C5/C6	0
C2	Ampl.2 harm.	1	C8	Ampl.4th harm.	1
C3	Max(u)	1	C9	C8/C5	0
C4	C2/C3	0	C10	Phase 2nd harm.	none
C5	L^2 norm of u	1	C11	Phase 3d harm.	none
C6	Var(u)	1			

Figure 1 Binary pictures



Samples of Patterns

Distorted Images

Table 5 Distances between shapes of the initial images (p1, p2,.., p8) and distorted ones (q1, q2,.., q8), using trace transform in 20×30 points.

	p1	p2	p3	p4	p5	p6	p7	p8
q1	0.503	1.546	1677	2.572	1.405	2.425	2.706	1.057
q2	1.671	0.417	2.224	3.423	1.678	1.690	2.711	1.554
q3	1.608	2.333	1.402	2.290	1.983	3.299	2.639	1.561
q4	2.469	3.182	1.487	0.585	2.794	4.772	4.079	2.499
q5	1.249	1.694	1.952	2.803	0.454	2.370	2.421	1.895
q6	2.575	1.791	3.602	4.579	1.935	0.778	1.965	3.305
q7	2.713	2.778	3.605	4.349	2.682	1.892	0.298	3.467
q8	1.442	1.534	1.299	2.154	1.944	3.090	3.764	0.640

Then we took triple features having ω (see (3)) being nonzero. The number of such triple features is greater. The results are presented in Table 6. It turns out that a problem of finding sizing coefficients is solved. In this Table 6, and below, we reduce the number of samples to 4 to show the numerical information in detail.

To find angles of rotation for the distorted images we use formulas from Table 1. There are two problems. First, these formulas are valid modulo τ. In our cases $\tau = \pi$ for C10 and $\tau = 2\pi/3$ for C11, see Table 7. To resolve this problem, we need both these functionals. For example, we note in the first three rows of Table 7 that $\Theta4$ is equal to -0.14 or 3.01. What value that is valid, we can find out using the three lines below. They provide us with the values which approximately equals 1, -1 or 3. This means that the proper value is 3.01, but not -0.14.

Another problem is that some triple features give wrong results. In the previous tasks we also got wrong results, however, the mean values were right. It is not good to take a mean value for angles. Therefore characteristics are needed which can approve a choice of triple features. In our examples we use phase angles of Fourier harmonics. Therefore we propose a coefficient "check"= (amplitude of the Fourier harmonic)/ (Variation of the underlying 2π-periodic function). Call it a check coefficient. It is computed for the samples of images and distorted images. They are in Table 7. To get the desired angle we want these two numbers (marked by "check" in Table 7) to be greater and approximately equaled. Such good pairs (two pairs for every trace functional) are pointed out by bold font in Table 7. The full variant of Table 7 must consist of 30 lines. We have chosen lines having the biggest coefficients marked by "check" in every column of the full variant of Table 7.

We take reliable values which have big "check," and use other values as an advice to establish k in the addend πk.

We see that angles are found properly.

For finding shifts we use theory of section 7. Results are in Table 8. Notice that five pairs (T1, D6), (T2, D6),..,(T5, D6) give the same result in view of the nature of these functionals. Thus there is only one row in Table 8 for these combinations. Let diagonals of each square on Fig.1 be equal to 200 units. Comparing Table 8 and Fig. 1, we see that shifts are found properly.

Table 6 Sizing coefficients

T	P and Φ	μ_1	μ_2	μ_3	μ_4
T1	24 combinations	1.087	0.985	0.730	0.876
T2	24 combinations	1.052	1.033	0.726	0.851
T3	24 combinations	1.105	0.984	0.747	0.813
T4	24 combinations	1.105	1.021	0.740	0.764
T5	24 combinations	1.106	0.996	0.734	0.808
Mean value		1.1	1.0	0.74	0.84

Table 7 Angles of rotation

Functionals			Angles of rotation for q1, q2, q3 and q4 and their validations (k=-1,0,1)											
T	P	Φ	θ_1	check		θ_2	check		θ_3	check		θ_4	check	
T3	D1	C10	**-0.32+πk**	89	91	**0.00+πk**	90	98	**-1.99+πk**	64	64	-0.11+πk	34	35
T4	D2	C10	-0.37+πk	43	39	0.04+πk	55	51	1.36+πk	21	19	**3.01+πk**	51	52
T5	D1	C10	-0.32+πk	54	67	-0.01+πk	74	72	1.15+πk	40	40	-0.14+πk	35	28
T2	D3	C11	-0.33+2πk/3	6	5	**0.01+2πk/3**	28	28	0.22+2πk/3	13	5	**0.86+2πk/3**	12	5
T5	D3	C11	-0.30+2πk/3	7	8	0.05+2πk/3	8	6	**0.10+2πk/3**	16	7	-1.18+2πk/3	10	3
T5	D6	C11	**-0.24+2πk/3**	15	17	0.01+2πk/3	20	11	0.18+2πk/3	12	4	0.91+2πk/3	5	2
Result			-0.32			0.00			-1.99			3.01		
True value			-0.30			0.00			-2.00			3.00		

Table 8 Computed shifts

Functionals		Shifts of sized and rotated samples p1..p4 to get q1..q4							
		p1 to q1		p2 to q2		p3 to q3		p4 to q4	
T	P	x	y	x	y	x	y	x	y
T1	D3	-0.14	-0.26	-6.74	-22.20	15.66	21.24	31.39	-8.37
T2	D3	-0.26	-0.87	-6.84	-22.44	15.31	21.15	31.30	-8.35
T3	D3	0.12	-0.16	-6.78	-22.14	15.83	21.14	31.52	-8.47
T4	D3	0.10	-0.42	-6.76	-22.05	15.75	21.28	31.47	-8.46
T5	D3	0.03	-0.29	-6.74	-22.14	15.69	21.28	31.43	-8.34
T1..T5	D6	-0.82	-0.72	-6.69	-21.88	15.81	20.97	31.41	-8.16
Result		0	0	-7	-22	16	21	31	-8

9 Image classification with triple features

Consider 5 classes a, b, g, e, and h of images presented on Fig.2. Every class on this picture is presented with 7 images. This pictorial information can be divided to 7 series, that is $\{a_1, b_1, g_1, e_1, h_1\}$ is the first series,.., $\{a_7, b_7, g_7, e_7, h_7\}$ is the seventh series. The task was to define the class of every image using other images. For example, let us fix series number j. We found numbers (they compose string headed "a_j" are in Table 9)

$$\sum_{i\neq j} dist(a_j,a_i),\ \sum_{i\neq j} dist(a_j,b_i),\ \sum_{i\neq j} dist(a_j,g_i),$$
$$\sum_{i\neq j} dist(a_j,e_i),\ \sum_{i\neq j} dist(a_j,h_i) \qquad (4)$$

they can be considered as distances between image a_j and the five classes, here every class is presented by six images (with exception of elements of the j-th series). The same distances can be found for every image in Fig. 2. Therefore we have 35 distances; they are in Table 9. The classifying process of the image a_j is a success if the least sum (4) is $\sum_{i\neq j} dist(a_j,a_i)$. The same is true for the all others images b_j, g_j, e_j and h_j.

Table 9 Finding Classes

	a	*b*	*g*	*e*	*h*
a_1	*1.03*	1.18	1.34	1.83	1.24
b_1	0.98	*0.82*	1.17	2.17	1.04
g_1	0.94	1.06	*0.84*	2.36	1.10
e_1	1.67	1.69	1.86	*1.21*	1.66
h_1	1.32	1.22	1.24	1.87	*0.93*
a_2	*0.84*	0.85	0.90	2.29	1.11
b_2	1.01	*0.82*	1.18	2.19	1.04
g_2	1.16	1.16	*0.99*	2.29	1.21
e_2	1.67	1.83	1.92	*1.17*	1.60
h_2	1.13	1.15	1.20	2.01	*0.97*
a_3	*0.79*	0.90	1.02	1.84	1.08
b_3	0.83	*0.78*	0.96	2.20	1.03
g_3	0.94	1.06	*0.80*	2.21	1.10
e_3	3.29	3.22	3.29	*1.72*	2.90
h_3	1.29	1.10	1.32	1.80	*0.84*
a_4	*0.82*	0.86	0.93	2.08	1.10
b_4	1.07	*0.91*	1.12	2.37	1.22
g_4	0.99	1.04	*0.81*	2.30	1.19
e_4	1.93	2.05	2.13	*0.95*	1.78
h_4	1.17	0.98	1.07	2.05	*0.97*
a_5	*0.76*	0.89	1.00	2.06	1.11
b_5	0.87	*0.72*	1.00	2.16	0.98
g_5	0.92	1.00	*0.79*	2.37	1.14
e_5	2.11	2.27	2.21	*1.11*	1.90
h_5	1.07	1.12	1.15	1.97	*0.91*
a_6	*0.79*	0.93	0.92	2.28	1.24
b_6	0.89	*0.82*	1.13	2.08	1.12
g_6	1.19	1.21	*1.00*	2.25	1.29
e_6	2.65	2.81	2.91	*1.34*	2.46
h_6	1.16	1.03	1.10	1.98	*0.90*
a_7	*0.82*	0.90	0.96	2.17	1.12
b_7	0.88	*0.83*	1.03	2.10	1.08
g_7	1.06	1.20	*0.92*	2.18	1.16
e_7	2.06	2.15	2.27	*1.07*	1.91
h_7	1.07	1.05	1.14	1.80	*0.92*

Every number in Table 9 was computed with 6×5×4=120 triple features defined by the following functionals. To compute the following trace functionals, numbers 2, 3, and 4, we made the images binary using threshold 5.5 in the color interval of 0..15. Trace functionals are 1) T1, 2) T2, 3) Number of segments in support 4) T3, 5) T4, 6)T5. Diametrical functionals are 1) D1, 2) D2, 3) D4, 4) D5, 5) Variation. Circus functionals are 1) C2/C3, 2) C5, 3) C5/C6, 4) C8/C5. Using results of Table 1, we see that all those 6×5×4 triple features are independent of any movement of images. Most of them are independent of sizing with exception of 6×5×1 features which use C5.

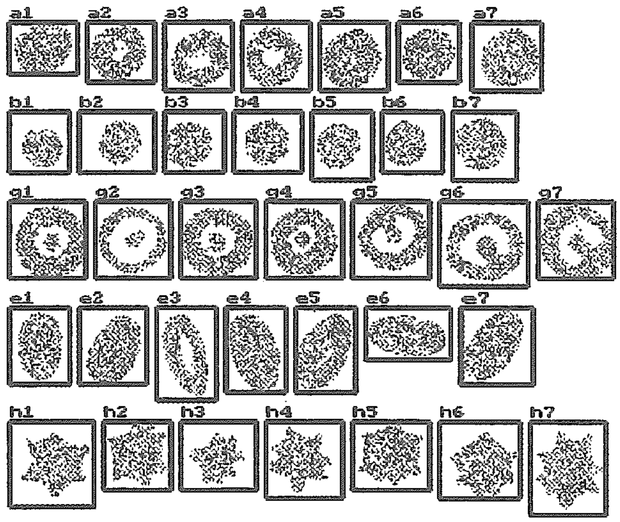
We also tried to fix the trace functional (one of the six), so we had 1×5×4 triple features and made six variants of recognition process. We saw that in these particular cases we were success in 80%. However, when we use the 6×5×4 triple features we are completely successful, as we see in Table 9.

Conclusion

The simple mathematical structure of triple features is described. A method of construction of a lot of invariant triple features is presented. These features are intended to be used in AI systems for image analyses.

We confirmed with our experiments that the theory of triple features is able to be used in systems for recognizing and clearing up parameters of rotation, sizing and displacement of images. Also, it is shown the theory is able to find classes of images.

Figure 2 Classes of Colored Images



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