

# A FUZZY INFORMATION RETRIEVAL METHOD BASED ON MULTI-RELATIONSHIP FUZZY CONCEPT NETWORKS

Yih-Jen Horng\*, Shyi-Ming Chen\*\*, and Chia-Hoang Lee\*

\*Department of Computer and Information Science  
National Chiao Tung University  
Hsinchu, Taiwan, R. O. C.

\*\*Department of Electronic Engineering  
National Taiwan University of Science and Technology  
Taipei, Taiwan, R. O. C.

## ABSTRACT

This paper presents a fuzzy information retrieval method based on multi-relationship fuzzy concept networks. There are six kinds of fuzzy relationships in a multi-relationship fuzzy concept network, i.e., “fuzzy positive association” relationship, “fuzzy negative association” relationship, “fuzzy kind of” relationship, “fuzzy instance of” relationship, “fuzzy superclass to” relationship, and “fuzzy classify” relationship. We use relevance matrices to model multi-relationship fuzzy concept networks. By calculating the transitive closure of relevance matrices, the implicit relevance degrees between concepts can be obtained. The satisfaction degrees that a document satisfies the user’s queries are then calculated when the concepts contained in a document and the concepts in the user’s query are related by different relationships. The users of fuzzy information retrieval systems could set different importance weights to these multi-relationship satisfaction degrees according to their needed information. The fuzzy information retrieval systems then aggregate these multi-relationship satisfaction degrees to find the most relevant documents with respect to the users’ queries. The proposed fuzzy information retrieval method can be more flexible than the existing methods.

## 1. INTRODUCTION

In [13], Lucarella et al. presented a fuzzy concept network structure which acts as a knowledge base for information retrieval. A fuzzy concept network consists of nodes and links. There are two kinds of nodes in a fuzzy concept network, i.e., documents and concepts. A link associated with a real value  $\mu$  between zero and one connecting two distinct concept nodes means these two concepts are relevant with strength  $\mu$ . A link with a real value  $\mu$  between zero and one connecting a concept node and a document node means that the content of this document contains the linked concept with strength  $\mu$ . Through the inference based on the links in the fuzzy concept network, the original concepts contained in the user’s query are expanded to derive some relevant concepts. By means of these expanded concepts, more documents can be retrieved. However, since the

inference process must be repeated every time when the users submit their query, the method of Lucarella et al. [13] for information retrieval is not efficient enough.

In [3] and [4], Chen et al. used concept matrices and document descriptor matrices to model fuzzy concept networks. The elements in a concept matrix describe the relevance degrees between concepts in a fuzzy concept network; the elements in a document descriptor matrix describe the strengths of the concepts in the fuzzy concept network belong to documents. By computing the transitive closure of concept matrices, the implicit relevance degrees between concepts that are not initially given by the domain experts are obtained. Since the inference is processed only once, the method proposed by Chen et al. for information retrieval is more efficient than the one proposed by Lucarella et al.

However, the methods proposed by Lucarella et al. [13] and Chen et al. [3], [4] are restricted since they all assumed that the concepts of fuzzy concept networks are linked by only one kind of fuzzy relationship [18], i.e., fuzzy positive association relationship. But for real world applications, there should be more than one relationship defined between concepts, e.g. synonym relationship, related-to relationship, generalization and specialization, just as the ones used in [2], [11], and [16]. In order to relieve the restriction of [3], [4], and [13], in [5] and [8] we extend the work of [3] and [4] to let the concepts be linked by one of the four fuzzy relationships, i.e., fuzzy positive association relationship, fuzzy negative association relationship, fuzzy generalization relationship, and fuzzy specialization relationship. Furthermore, we also proposed some methods to deal with users’ queries based on this kind of fuzzy concept networks in the information retrieval system.

There are still some drawbacks in [5] and [8]. In [5] and [8], we assumed that there is only one kind of fuzzy relationship between any pair of the concept. But as Miyamoto stated in [14], there are no theoretical reasons for avoiding multiple defined relationships between concepts. Because the relationships between concepts may diverse in different contexts, the concept pair should have different relationships at the same time, where each has its own strength degree. If we can let the concept pairs in fuzzy concept networks have multiple relationships simultaneously, then there is room for

more flexibility in fuzzy information retrieval systems.

In this paper, we extend the works of [5] and [8] to allow multiple fuzzy relationships between each pair of concepts at the same time in fuzzy concept networks, where each relationship has its own linking strength. Moreover, we use six fuzzy relationships [11], [12] (i.e., “fuzzy positive association” relationship, “fuzzy negative association” relationship, “fuzzy kind of” relationship, “fuzzy instance of” relationship, “fuzzy superclass to” relationship, and “fuzzy classify” relationship) to describe the relationships between concepts in this kind of fuzzy concept networks. If the linking strength between two concepts by a specified relationship  $r$  is not explicitly given by experts, they can be inferred by means of other links by the same relationship  $r$ . The users of the fuzzy information retrieval system can find more relevant documents containing not only the concepts in the users’ queries but also the related concepts by some important relationships. They can assign the different importance degrees with respect to different fuzzy relationship in three ways: by setting different importance weights to different relationships, respectively, by setting an importance order of the relationships, and by using some pre-defined simple linguistic quantifiers [1]. The fuzzy information retrieval system then aggregates these multi-relationship relevance degrees to obtain overall satisfaction degrees between documents and the users’ queries and therefore find the most relevant documents.

In this paper, we adopt the IOWA (Induced Ordered Weighted Averaging) operators [16] as the method of aggregating multi-relationship satisfaction degrees between documents and the users’ queries. Since the multi-relationship satisfaction degrees between documents and the users’ queries can be indexed by the relationships’ name, the IOWA operators are suitable to be used in the fuzzy information retrieval systems.

Since the concept pairs in the multi-relationship fuzzy concept networks could have multiple relationships simultaneously, the proposed fuzzy information retrieval method can be more flexible than the existing methods.

## 2. MULTI-RELATIONSHIP FUZZY CONCEPT NETWORKS

In a multi-relationship fuzzy concept network, the concepts are related to other concepts by more than one relationship at the same time, each has its own relevance degree. In this paper, we assume that the relevance degrees between concepts are specified by domain experts. There are six kinds of fuzzy relationships between concepts in a multi-relationship fuzzy concept network, which are described as follows:

(1) Fuzzy positive association [11]:

It relates concepts which have a fuzzy similar meaning in some contexts.

(2) Fuzzy negative association [11]:

It relates concepts which are fuzzy complementary, fuzzy incompatible or fuzzy antonyms.

(3) Fuzzy kind of [12]:

A concept is regarded as a fuzzy kind of another concept if it is a specialization or a subclass of that concept.

(4) Fuzzy instance of [12]:

A concept is regarded as a fuzzy instance of another concept if it partially belongs to that concept.

(5) Fuzzy superclass to [12]:

It is the inverse of the “fuzzy kind of” relationship.

(6) Fuzzy classify [12]:

It is the inverse of the “fuzzy instance of” relationship.

The fuzzy relationships between concepts described above are summarized as follows.

**Definition 2.1:** Let  $C$  be a set of concepts. Then,

(1) “Fuzzy positive association”  $P$  is a fuzzy relation,  $P: C \times C \rightarrow [0, 1]$ , which is reflexive, symmetric, and max- $*$ -transitive.

(2) “Fuzzy negative association”  $N$  is a fuzzy relation,  $N: C \times C \rightarrow [0, 1]$ , which is anti-reflexive, symmetric, and max- $*$ -nontransitive.

(3) “Fuzzy kind of”  $K$  is a fuzzy relation,  $K: C \times C \rightarrow [0, 1]$ , which is anti-reflexive, anti-symmetric, and max- $*$ -transitive.

(4) “Fuzzy instance of”  $I$  is a fuzzy relation,  $I: C \times C \rightarrow [0, 1]$ , which is anti-reflexive, anti-symmetric, and max- $*$ -transitive.

(5) “Fuzzy superclass to”  $S$  is a fuzzy relation,  $S: C \times C \rightarrow [0, 1]$ , which is anti-reflexive, anti-symmetric, and max- $*$ -transitive.

(6) “Fuzzy classify”  $A$  is a fuzzy relation,  $A: C \times C \rightarrow [0, 1]$ , which is anti-reflexive, anti-symmetric, and max- $*$ -transitive.

**Definition 2.2:** A multi-relationship fuzzy concept network is denoted as  $MRFCN(E, L)$ , where  $E$  is a set of nodes, and each node stands for a concept or a document;  $L$  is a set of directed edges between nodes. If  $\ell \in L$ , then the directed edge  $\ell$  has the following two formats:

(1)  $c_i \xrightarrow{(\langle \mu_P, P \rangle, \langle \mu_N, N \rangle, \langle \mu_K, K \rangle, \langle \mu_I, I \rangle, \langle \mu_S, S \rangle, \langle \mu_A, A \rangle)} c_j$ , which

means the directed edge  $\ell$  connects from concept  $c_i$  to concept  $c_j$  with a six-tuple

$(\langle \mu_P, P \rangle, \langle \mu_N, N \rangle, \langle \mu_K, K \rangle, \langle \mu_I, I \rangle, \langle \mu_S, S \rangle, \langle \mu_A, A \rangle)$ ,

where  $\mu_P$  indicates the relevance degree of “fuzzy positive association” relationship  $P$  between concept  $c_i$  and concept  $c_j$  (i.e., there is a “fuzzy positive association” relationship between concept  $c_i$  and concept  $c_j$  with degree  $\mu_P$ ),  $\mu_N$  indicates the relevance degree of “fuzzy negative association” relationship  $N$  between concept  $c_i$  and concept  $c_j$  (i.e., there is a “fuzzy negative association” relationship between concept  $c_i$  and concept  $c_j$  with degree  $\mu_N$ ),  $\mu_K$  indicates the relevance degree of “fuzzy kind of” relationship  $K$  between concept  $c_i$  and concept  $c_j$  (i.e., concept  $c_i$  is a kind of concept  $c_j$  with degree  $\mu_K$ ),  $\mu_I$  indicates the relevance degree of “fuzzy

instance of” relationship  $I$  between concept  $c_i$  and concept  $c_j$  (i.e., concept  $c_i$  is an instance of concept  $c_j$  with degree  $\mu_I$ ),  $\mu_S$  indicates the relevance degree of “fuzzy superclass to” relationship  $S$  between concept  $c_i$  and concept  $c_j$  (i.e., concept  $c_i$  is a superclass of concept  $c_j$  with degree  $\mu_S$ ),  $\mu_A$  indicates the relevance degree of “fuzzy classify” relationship  $A$  between concept  $c_i$  and concept  $c_j$  (i.e., concept  $c_i$  classify concept  $c_j$  with degree  $\mu_A$ ),  $\mu_P \in [0,1]$ ,  $\mu_N \in [0,1]$ ,  $\mu_K \in [0,1]$ ,  $\mu_I \in [0,1]$ ,  $\mu_S \in [0,1]$ , and  $\mu_A \in [0,1]$ .

(2)  $c_i \xrightarrow{(\langle \mu_P, P \rangle, \langle \mu_N, N \rangle, \langle \mu_K, K \rangle, \langle \mu_I, I \rangle, \langle \mu_S, S \rangle, \langle \mu_A, A \rangle)} d_j$ , which

means the directed edge  $\ell$  connects from concept  $c_i$  to document  $d_j$  with a six-tuple

$(\langle \mu_P, P \rangle, \langle \mu_N, N \rangle, \langle \mu_K, K \rangle, \langle \mu_I, I \rangle, \langle \mu_S, S \rangle, \langle \mu_A, A \rangle)$ ,

where  $\mu_P$  indicates the relevance degree of “fuzzy positive association” relationship  $P$  between concept  $c_i$  and document  $d_j$  (i.e., there is a “fuzzy positive association” relationship between concept  $c_i$  and the concepts contained in document  $c_j$  with degree  $\mu_P$ ),  $\mu_N$  indicates the relevance degree of “fuzzy negative association” relationship  $N$  between concept  $c_i$  and document  $d_j$  (i.e., there is a “fuzzy negative association” relationship between concept  $c_i$  and the concepts contained in document  $c_j$  with degree  $\mu_N$ ),  $\mu_K$  indicates the relevance degree of “fuzzy kind of” relationship  $K$  between concept  $c_i$  and document  $d_j$  (i.e., concept  $c_i$  is a kind of the concepts contained in document  $c_j$  with degree  $\mu_K$ ),  $\mu_I$  indicates the relevance degree of “fuzzy instance of” relationship  $I$  between concept  $c_i$  and document  $d_j$  (i.e., concept  $c_i$  is an instance of the concepts contained in document  $c_j$  with degree  $\mu_I$ ),  $\mu_S$  indicates the relevance degree of “fuzzy superclass to” relationship  $S$  between concept  $c_i$  and document  $d_j$  (i.e., concept  $c_i$  is a superclass of the concepts contained in document  $c_j$  with degree  $\mu_S$ ),  $\mu_A$  indicates the relevance degree of “fuzzy classify” relationship  $A$  between concept  $c_i$  and document  $d_j$  (i.e., concept  $c_i$  classify the concepts contained in document  $c_j$  with degree  $\mu_A$ ),  $\mu_P \in [0,1]$ ,  $\mu_N \in [0,1]$ ,  $\mu_K \in [0,1]$ ,  $\mu_I \in [0,1]$ ,  $\mu_S \in [0,1]$ , and  $\mu_A \in [0,1]$ .

**Example 2.1:** Assume there is a multi-relationship fuzzy concept network as shown in Fig. 1, where  $c_1, c_2, \dots, c_7$  are concepts, and  $d_1, d_2, d_3$  and  $d_4$  are documents.

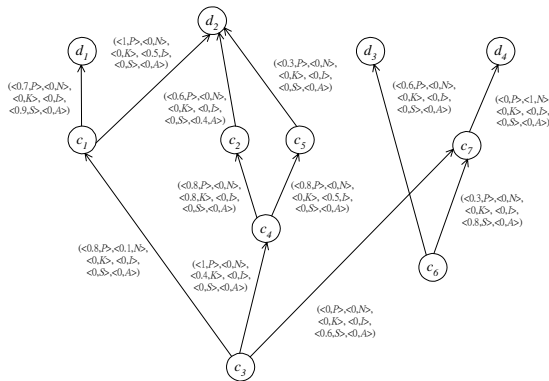


Fig. 1. A multi-relationship fuzzy concept network.

From Fig. 1, we can see that the contents of document  $d_2$  have 100% (relevance degree = 1) “fuzzy positive association” relationship with respect to concept  $c_i$ ; the contents of document  $d_2$  have 50% (relevance degree = 0.5) “fuzzy instance of” relationship with respect to concept  $c_i$ ; but the contents of document  $d_2$  and concept  $c_i$  are not related by “fuzzy negative association” relationship, “fuzzy kind of” relationship, “fuzzy superclass to” relationship, and “fuzzy classify” relationship. Concept  $c_1$  has 80% (relevance degree = 0.8) “fuzzy positive association” relationship with respect to concept  $c_3$ ; concept  $c_1$  has 10% (relevance degree = 0.1) “fuzzy negative association” relationship with respect to concept  $c_3$ ; but concept  $c_1$  and concept  $c_3$  are not related by “fuzzy kind of” relationship, “fuzzy instance of” relationship, “fuzzy superclass to” relationship, and “fuzzy classify” relationship.

However, the domain experts may forget to set the relevance degrees between concepts. In this case, the relevance degree between concepts may be inferred by some intermediate links between them. In a multi-relationship fuzzy concept network, if the fuzzy relationship  $r$  is transitive, i.e.,  $r \in \{P, K, I, S, A\}$ , and the relevance degree between node  $e_i$  and node  $e_j$  related by fuzzy relationship  $r$  is  $\mu_{ij}^r$ , where  $\mu_{ij}^r \in [0, 1]$ , and if the relevance degree between node  $e_j$  and node  $e_k$  related by fuzzy relationship  $r$  is  $\mu_{jk}^r$ , where  $\mu_{jk}^r \in [0, 1]$ , then the relevance degree  $\mu_{ik}^r$  between node  $e_i$  and node  $e_k$  related by fuzzy relationship  $r$  can be inferred by the following expression:

$$\mu_{ik}^r = \text{Min}(\mu_{ij}^r, \mu_{jk}^r), \quad (1)$$

where  $\mu_{ik}^r \in [0, 1]$ . Furthermore, if the relevance degree between node  $e_1$  and node  $e_2$  by fuzzy relationship  $r$  is  $\mu_{12}^r$ , the relevance degree between node  $e_2$  and node  $e_3$  by fuzzy relationship  $r$  is  $\mu_{23}^r, \dots$ , and the relevance degree between node  $e_{n-1}$  and node  $e_n$  by fuzzy relationship  $r$  is  $\mu_{(n-1)n}^r$ , where  $\mu_{12}^r \in [0, 1]$ ,  $\mu_{23}^r \in [0, 1], \dots$ , and  $\mu_{(n-1)n}^r \in [0, 1]$ , then the relevance degree between node  $e_1$  and node  $e_n$  by fuzzy relationship  $r$  is  $\mu_{1n}^r$ , where  $\mu_{1n}^r \in [0, 1]$  and

$$\mu_{1n}^r = \text{Min}(\mu_{12}^r, \mu_{23}^r, \dots, \mu_{(n-1)n}^r). \quad (2)$$

It means that if there is a route that started from node  $e_1$  and ended at node  $e_n$ , then the relevance degree between node  $e_1$  and node  $e_n$  by fuzzy relationship  $r$  is dominated by the weakest link. However, if there are  $h$  routes between node  $e_1$  and node  $e_n$ , then the actual relevance degree between node  $e_1$  and node  $e_n$  by the fuzzy relationship  $r$  can be calculated by the following formula:

$$\mu_{1n}^r = \text{Max}(\mu_{1n}^{r(1)}, \mu_{1n}^{r(2)}, \dots, \mu_{1n}^{r(h)}), \quad (3)$$

where  $\mu_{1n}^{r(i)}$  is the  $i$ th route that started from node  $e_i$  and ended at node  $e_n$ , and  $1 \leq i \leq h$ .

In the proposed multi-relationship fuzzy concept network, we don't allow cross-relationship inferences due to the fact that they need complicated mechanisms to decide the resulting relationship induced by two different relationships but the resulting relationship may not be always right.

In a multi-relationship fuzzy concept network, each user's query can be represented by a query descriptor  $Q$  expressed as a fuzzy subset of the collection of concepts by the following expression:

$$Q = \{(c_i, f_Q(c_i)) \mid c_i \in C\},$$

where  $f_Q(c_i)$ ,  $f_Q : C \rightarrow [0, 1]$ , represents the relevance value of the query descriptor  $Q$  with respect to the concept  $c_i$ , i.e., the strength that a user thinks concept  $c_i$  should be contained in the retrieved documents. By means of the multi-relationship links between concept pairs and between concepts and documents, the original meanings of each user's query can be expanded to contain more related concepts by some specified fuzzy relationships. Thus, we can retrieve more documents containing concepts that are not specified but are somehow related to the original user's query.

The relevance degrees between documents and concepts related by different fuzzy relationships should then be aggregated to obtain the overall relevance degrees between documents and concepts. We decide to take a dynamic approach to let the user control the aggregation operation by giving different importance weights to the relevance degrees between documents and concepts related by different fuzzy relationships. This approach seems more suitable than statically averaging those relevance degrees since it allows the users to express their needed information by setting different importance weights to the relevance degrees between documents and concepts related by different fuzzy relationships.

In this paper, the IOWA aggregation operators are utilized to obtain the overall relevance degrees between documents and concepts. The arguments of the IOWA aggregation operators are weighted according to their indexing values before aggregated. If we index the relevance degrees between documents and concepts by the fuzzy relationships' name, then the IOWA operators could satisfy our application.

### 3. IOWA AGGREGATION OPERATORS

In [15], Yager proposed a family of mean-like operators which are used to deal with multicriteria decisionmaking problems. The arguments of these operators are weighted according to their order made by

sorting the arguments and then averaged according to their weights, so these operators are named OWA (Ordered Weighted Averaging) operators. By giving different weighting vectors, the OWA operators are lying between choosing the minimum and choosing the maximum of the arguments. We briefly review some definitions of the OWA operators [15] as follows.

**Definition 3.1:** An OWA operator that has  $n$  input arguments is a mapping

$$F: R^n \rightarrow R,$$

which has a weighting vector  $W$  of dimension  $n$  associated with it. The weighting vector  $W$  has the following properties:

$$w_j \in [0, 1],$$

$$\sum_{j=1}^n w_j = 1,$$

and such that

$$F(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j, \quad (4)$$

where  $b_j$  is the  $j$ th largest value of the input arguments  $a_1, a_2, \dots$ , and  $a_n$ , and  $1 \leq j \leq n$ .

If  $B$  is a vector consisting of the ordered arguments  $a_i$ , which is called the ordered argument vector, and  $W^T$  is the transpose of the weighting vector, then the OWA aggregation can also be expressed as:

$$F_w(a_1, a_2, \dots, a_n) = W^T B. \quad (5)$$

The OWA operators are also applied in the information retrieval field. In [6], Damiani et al. proposed a fuzzy retrieval model to retrieve reusable components containing the features needed by users. Since each feature contributes different weights to components under different contexts (categories), they used the OWA operator associated with *context weights* to obtain the overall weights of the reusable components with respect to the features needed by the user. In [1], Bordogna et al. proposed a document retrieval model where the documents are divided into subparts. They used the OWA operator to aggregate the significances of the term with respect to different subparts of the document by some linguistic quantifiers which are formalized as corresponding weighting vectors.

In [16], Yager et al. proposed a more general form of the OWA operator, which is called IOWA operators since the aggregating operation is controlled by the order inducing variable. The IOWA operators aggregate the two-tuples  $\langle u_i, a_i \rangle$ , which are denoted as *OWA pairs*, where  $u_i$  the order inducing variable and  $a_i$  is the argument variable. The IOWA aggregation of the OWA pairs is calculated as follows.

$$F_w(\langle u_1, a_1 \rangle, \dots, \langle u_n, a_n \rangle) = W^T B_u, \quad (6)$$

where  $B_u$  is an ordered argument vector and  $W$  is a weighting vector. The  $b_j$  in the ordered argument vector  $B_u$  is the  $a$  value of the OWA pair having the  $j$ th largest  $u$

value. If we let the order inducing variable  $u_i$  be equal to argument variable  $a_i$  then the IOWA operators are the same as OWA operators, since the order of  $u_i$  is the same as the order of  $a_i$ . Moreover, the value of the order inducing variable  $u_i$  can be not only real numbers but also any values that have a linear ordering.

The argument variables  $a_i$  of the IOWA aggregation operators are weighted according to their indexing values (i.e., the order inducing variable  $u_i$ ). If we index the relevance degrees between documents and concepts by the fuzzy relationships' name, and give these indexes a specified ordering, then the IOWA operators could satisfy our needed application. In the next section, we propose a fuzzy query processing method for document retrieval based on multi-relationship fuzzy concept networks.

#### 4. FUZZY QUERY PROCESSING FOR DOCUMENT RETRIEVAL BASED ON MULTI-RELATIONSHIP FUZZY CONCEPT NETWORKS

In this paper, we use six relevance matrices to represent the relevance degrees between concepts in a multi-relationship fuzzy concept network since there are six fuzzy relationships defined in a multi-relationship fuzzy concept network. Each relevance matrix describes the relevance degrees between concepts, where the concepts are related by one kind of fuzzy relationship. By computing the transitive closure of these relevance matrices, the implicit relevance degrees between concepts can be obtained. The definitions of relevance matrices and their transitive closures are described as follows.

**Definition 4.1:** A relevance matrix  $V_r$  is a fuzzy matrix [10], where the element  $V_r(c_i, c_j)$  represents the relevance degree between concept  $c_i$  and concept  $c_j$  when they are connected by fuzzy relationship  $r$ ,  $V_r(c_i, c_j) \in [0, 1]$ , and  $r \in \{P, N, K, I, S, A\}$ . If fuzzy relationship  $r$  is reflexive then  $V_r(c_i, c_i) = 1$ , else  $V_r(c_i, c_i) = 0$ . If fuzzy relationship  $r$  is symmetric then  $V_r(c_i, c_j) = V_r(c_j, c_i)$ . If  $V_r(c_i, c_j) = 0$ , then the relevance degree between concept  $c_i$  and concept  $c_j$  is not defined explicitly by the experts.

**Definition 4.2:** Assume that  $V_r$  is a relevance matrix,  $r \in \{P, N, K, I, S, A\}$ , and

$$V_r = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1} & v_{n2} & \cdots & v_{nn} \end{bmatrix},$$

where  $n$  is the number of concepts,  $v_{ij} \in [0, 1]$ ,  $1 \leq i \leq n$ , and  $1 \leq j \leq n$ . If fuzzy relationship  $r$  is nontransitive, then we let the transitive closure of  $V_r$  be  $V_r^*$ , i.e.,  $V_r^* = V_r$ . If fuzzy relationship  $r$  is transitive, then the transitive closure  $V_r^*$  is defined as follows. Let

$$V_r^2 = V_r \otimes V_r,$$

$$= \begin{bmatrix} \bigvee_{i=1, \dots, n} (v_{i1} \wedge v_{i1}) & \bigvee_{i=1, \dots, n} (v_{i1} \wedge v_{i2}) & \cdots & \bigvee_{i=1, \dots, n} (v_{i1} \wedge v_{in}) \\ \bigvee_{i=1, \dots, n} (v_{2i} \wedge v_{i1}) & \bigvee_{i=1, \dots, n} (v_{2i} \wedge v_{i2}) & \cdots & \bigvee_{i=1, \dots, n} (v_{2i} \wedge v_{in}) \\ \vdots & \vdots & \ddots & \vdots \\ \bigvee_{i=1, \dots, n} (v_{ni} \wedge v_{i1}) & \bigvee_{i=1, \dots, n} (v_{ni} \wedge v_{i2}) & \cdots & \bigvee_{i=1, \dots, n} (v_{ni} \wedge v_{in}) \end{bmatrix}, \quad (7)$$

where “ $\vee$ ” is the maximum operator and “ $\wedge$ ” is the minimum operator. Then, according to [2], the transitive closure  $V_r^*$  of  $V_r$  is defined as:

$$V_r^* = V_r \cup V_r^{n-1}, \quad (8)$$

where “ $\cup$ ” is the union operator,  $V_r^k$  is calculated recursively,  $V_r^k = V_r \cup V_r^{k-1}$ , and the powers on  $V_r$  are computed by formula (7).

Moreover, we use six document descriptor matrices to represent the relevance degrees between concepts and documents of the multi-relationship fuzzy concept network. Each relevance matrix describes the relevance degrees between concepts and documents, where the concepts and documents are related by one kind of fuzzy relationship.

**Definition 4.3:** Let  $D$  be a set of documents,  $D = \{d_1, d_2, \dots, d_m\}$ , and let  $C$  be a set of concepts,  $C = \{c_1, c_2, \dots, c_n\}$ . The document descriptor matrix  $P_r$  is shown as follows:

$$P_r = \begin{matrix} & c_1 & c_2 & \cdots & c_n \\ \begin{matrix} d_1 \\ d_2 \\ \vdots \\ d_m \end{matrix} & \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{bmatrix} \end{matrix},$$

where  $m$  is the number of documents,  $n$  is the number of concepts,  $p_{ij}$  stands for the relevance degree between document  $d_i$  and concept  $c_j$  related by fuzzy relationship  $r$  (i.e., the relevance degree between the content of document  $d_i$  and concept  $c_j$  related by fuzzy relationship  $r$ ),  $p_{ij} \in [0, 1]$ , where  $r \in \{P, N, K, I, S, A\}$ ,  $1 \leq i \leq m$ , and  $1 \leq j \leq n$ .

In a document descriptor matrix  $P_r$ , the elements are given explicitly by domain experts to denote the relevance degree between concepts and documents when they are related by fuzzy relationship  $r$ ,  $r \in \{P, N, K, I, S, A\}$ . However, the experts may forget to set the relevance degrees between some concepts and documents. In this case, we can obtain the implicit relevance degrees by means of the following formula:

$$\begin{cases} P_r^* = P_r \cup (V_r^* \otimes P_r) & \text{if } r \text{ is transitive} \\ P_r^* = P_r & \text{if } r \text{ is nontransitive,} \end{cases} \quad (9)$$

where  $V_r^*$  is the transitive closure of the relevance matrix  $V_r$ , and  $r \in \{P, N, K, I, S, A\}$ .  $P_r^*$  is referred to as an expanded document descriptor matrix of document descriptor matrix  $P_r$ . Since there are six kinds of fuzzy relationships defined in the multi-relationship fuzzy concept network, we can obtain six expanded document descriptor matrices. These six expanded document descriptor matrices are then used as a basis for similarity measures between users' queries and documents.

The user's query  $Q$  can be represented by a query

descriptor vector  $\bar{q}$ . In this case, if the user's query is as follows:

$$Q = \{(c_1, x_1), (c_2, x_2), \dots, (c_n, x_n)\},$$

then

$$\bar{q} = \langle x_1, x_2, \dots, x_n \rangle,$$

where  $x_i \in [0, 1]$  indicates the desired relevance degree of the document with respect to concept  $c_i$ , and  $1 \leq i \leq n$ .

In a query descriptor relevance vector  $\bar{q}$ , if  $x_i = 0$ , then it indicates that documents desired by the user don't possess concept  $c_i$ . If  $x_i = "-"$ , then it indicates that the relevance degree of the desired documents with respect to concept  $c_i$  can be neglected.

Let  $x$  and  $y$  be two values where  $x \in [0, 1]$ ,  $y \in [0, 1]$ , then the degree of similarity between  $x$  and  $y$  can be evaluated by the function  $T$  [3],

$$T(x, y) = 1 - |x - y|, \quad (10)$$

where  $T(x, y) \in [0, 1]$ . The larger the value of  $T(x, y)$ , the more the similarity between  $x$  and  $y$ .

Assume that the document descriptor relevance vector  $\bar{dr}_i$  (i.e., the  $i$ th row of the expanded document descriptor relevance matrix  $P_r^*$ , where  $r \in \{P, N, K, I, S, A\}$ ), and the query descriptor relevance vector  $\bar{q}$  are represented as follows:

$$\begin{aligned} \bar{dr}_i &= \langle s_{i1}, s_{i2}, \dots, s_{in} \rangle, \\ \bar{q} &= \langle x_1, x_2, \dots, x_n \rangle, \end{aligned}$$

where  $s_{ij} \in [0, 1]$ ,  $x_i \in [0, 1]$ ,  $1 \leq j \leq n$ ,  $1 \leq i \leq m$ ,  $n$  is the number of concepts, and  $m$  is the number of documents, and  $r \in \{P, N, K, I, S, A\}$ . Let  $\bar{q}(j)$  be the  $j$ th element of the query descriptor relevance vector  $\bar{q}$ .

If  $\bar{q}(j) = "-"$ , then it indicates that concept  $c_j$  is neglected by the user's query. The degree of satisfaction  $DS(d_i)$  that document  $d_i$  satisfies the user's query  $Q$  by fuzzy relationship  $r$  can be evaluated by [3]:

$$DS(d_i) = \frac{\sum_{\substack{q^{(j)} \neq "-" \text{ and } j=1, \dots, n}} T(s_{ij}, x_j)}{k}, \quad (11)$$

where  $DS(d_i) \in [0, 1]$ ,  $1 \leq i \leq m$ , and  $k$  is the number of concepts not neglected by the user's query. The larger the value of  $DS(d_i)$ , the more the degree of satisfaction that the document  $d_i$  satisfies the user's query by fuzzy relationship  $r$ ,  $r \in \{P, N, K, I, S, A\}$ .

The degrees of satisfaction that the document satisfies the user's query by different fuzzy relationships are then aggregated to obtain the overall satisfaction that the document satisfies the user's query by utilizing the IOWA aggregation operators. The users could control the IOWA aggregation operations to express their preferred fuzzy relationships by setting different weighting vectors associated to the IOWA aggregation operators. They can do this in three ways: by setting different importance weights to the relationships, by

setting an importance ordering of the fuzzy relationships, and by using some predefined linguistic quantifiers. We describe these approaches as follows.

Let  $D$  be a set of documents,  $D = \{d_1, d_2, \dots, d_m\}$ , and let  $C$  be a set of concepts,  $C = \{c_1, c_2, \dots, c_n\}$ , defined in a multi-relationship fuzzy concept network. The degrees of satisfaction that the document  $d_i$  satisfies the user's query by fuzzy relationships  $r$  are denoted as  $DS_r(d_i)$ , where  $1 \leq i \leq m$  and  $r \in \{P, N, K, I, S, A\}$ .

**Case 1:** If the users assign the importance weights directly to each degree of satisfaction that the document satisfies the user's query by different fuzzy relationships, then we let the names of the fuzzy relationships be the inducing order variables of the IOWA operators, and have the following ordering:

$$P > N > K > I > S > A,$$

and let  $DS_r(d_i)$  be the argument variable of the IOWA operators. That is, the *OWA pairs* of the IOWA operators are represented as  $\langle r, DS_r(d_i) \rangle$ , where  $1 \leq i \leq m$  and  $r \in \{P, N, K, I, S, A\}$ . Assume that the user assigns the importance weight  $w_r$  to the degree of satisfaction that the document satisfies the user's query by fuzzy relationships  $r$ , where  $0 \leq w_r \leq 1$  and  $r \in \{P, N, K, I, S, A\}$ . In order to satisfy the IOWA operators' properties, the summation of these importance weights must be equal to one (i.e.,  $w_P + w_N + w_K + w_I + w_S + w_A = 1$ ). Then, the weighting vector  $W$  of the IOWA operator is

$$W = \begin{bmatrix} w_P \\ w_N \\ w_K \\ w_I \\ w_S \\ w_A \end{bmatrix}.$$

After performing the ordering process to these OWA pairs according to the ordering inducing variable, we can get the ordered argument vector  $B$  shown as follows:

$$B = \begin{bmatrix} DS_P(d_i) \\ DS_N(d_i) \\ DS_K(d_i) \\ DS_I(d_i) \\ DS_S(d_i) \\ DS_A(d_i) \end{bmatrix},$$

and the overall degree of satisfaction  $DS(d_i)$  that the document  $d_i$  satisfies the user's query can be calculated shown as follows:

$$\begin{aligned} DS(d_i) &= F_W(\langle P, DS_P(d_i) \rangle, \langle N, DS_N(d_i) \rangle, \langle K, DS_K(d_i) \rangle, \langle I, DS_I(d_i) \rangle, \langle S, DS_S(d_i) \rangle, \langle A, DS_A(d_i) \rangle) \\ &= w_P \times DS_P(d_i) + w_N \times DS_N(d_i) + w_K \times DS_K(d_i) + w_I \times DS_I(d_i) + w_S \times DS_S(d_i) + w_A \times DS_A(d_i) \end{aligned} \quad (12)$$

**Case 2:** If the users give an importance weights order of the degrees of satisfaction that the document satisfies the user's query by different fuzzy relationships, then

we let the name of each fuzzy relationship be the inducing order variable of the IOWA operators, and have the following user-given ordering:

$$r_1 > r_2 > \dots > r_6,$$

where  $r_i \in \{P, N, K, I, S, A\}$ ,  $r_1 \neq r_2 \neq \dots \neq r_6$ ,  $1 \leq i \leq 6$ , and let  $DS_r(d_i)$  be the argument variable of the IOWA operators. That is, the OWA pairs of the IOWA operators are represented as  $\langle r, DS_r(d_i) \rangle$ , where  $1 \leq i \leq m$  and  $r \in \{P, N, K, I, S, A\}$ . The weighting vector  $W$  of the IOWA operator is

$$W = \begin{bmatrix} 6/21 \\ 5/21 \\ 4/21 \\ 3/21 \\ 2/21 \\ 1/21 \end{bmatrix}.$$

After performing the ordering process to these OWA pairs according to the ordering inducing variable, we can get the ordered argument vector  $B$  shown as follows:

$$B = \begin{bmatrix} DS_{r_1}(d_i) \\ DS_{r_2}(d_i) \\ \vdots \\ DS_{r_6}(d_i) \end{bmatrix},$$

where  $r_i \in \{P, N, K, I, S, A\}$ ,  $r_1 \neq r_2 \neq \dots \neq r_6$ ,  $1 \leq i \leq 6$ , and the overall degree of satisfaction  $DS(d_i)$  that the document  $d_i$  satisfies the user's query can be calculated shown as follows:

$$\begin{aligned} DS(d_i) &= F_W(\langle P, DS_P(d_i) \rangle, \langle N, DS_N(d_i) \rangle, \langle K, DS_K(d_i) \rangle, \langle I, DS_I(d_i) \rangle, \langle S, DS_S(d_i) \rangle, \langle A, DS_A(d_i) \rangle) \\ &= \frac{6}{21} \times DS_P(d_i) + \frac{5}{21} \times DS_N(d_i) + \frac{4}{21} \times DS_K(d_i) + \frac{3}{21} \times DS_I(d_i) + \frac{2}{21} \times DS_S(d_i) + \frac{1}{21} \times DS_A(d_i), \end{aligned} \quad (13)$$

where  $r_i \in \{P, N, K, I, S, A\}$ ,  $r_1 \neq r_2 \neq \dots \neq r_6$ , and  $1 \leq i \leq 6$ .

**Case 3:** If the users give a predefined linguistic quantifier which can be formalized as a corresponding weighting vector, then we let  $DS_r(d_i)$ ,  $1 \leq i \leq m$ ,  $r \in \{P, N, K, I, S, A\}$ , be the inducing order variable and the argument variable of the IOWA operators at the same time. That is, the OWA pairs of the IOWA operators are represented as  $\langle DS_r(d_i), DS_r(d_i) \rangle$ , where  $1 \leq i \leq m$ , and  $r \in \{P, N, K, I, S, A\}$ . Based on [1], we defined four linguistic quantifiers, i.e., "all relationships", "at least one relationship", "at least  $t$  relationships", and "at least  $t$  percent relationships" in the proposed multi-relationship fuzzy information retrieval system. If the linguistic quantifier given by the user is "all relationships", then it means that all relationships are considered important to compute the relevance degree between the documents and the users' queries. Then, the corresponding weighting vector  $W$  is:

$$W = \begin{bmatrix} 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \\ 1/6 \end{bmatrix}.$$

If the linguistic quantifier given by the user is "at least one relationship", then it means that only the relationship with the largest relevance degree between the documents and users' queries is considered. Then, the corresponding weighting vector  $W$  is:

$$W = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

If the linguistic quantifier given by the user is "at least  $t$  relationships", then it means that only the relationships within the top  $t$ th relevance degree between the documents and users' queries are considered. Then, the corresponding weighting vector  $W$  is:

$$W = \begin{bmatrix} w_1 & 1/t \\ \vdots & \vdots \\ w_t & 1/t \\ w_{t+1} & 0 \\ \vdots & \vdots \\ w_6 & 0 \end{bmatrix},$$

where (1)  $w_j = 1/t$  when  $1 \leq j \leq t$ , (2)  $w_j = 0$  when  $t < j \leq 6$ , where  $1 \leq j, t \leq 6$ . If the linguistic quantifier given by the user is "at least  $t$  percent relationships", then it means that only the relationships within the top  $t$  percents relevance degrees between the documents and users' queries are considered. Then, the corresponding weighting vector  $W$  is:

$$W = \begin{bmatrix} w_1 & 1/l \\ \vdots & \vdots \\ w_l & 1/l \\ w_{l+1} & 0 \\ \vdots & \vdots \\ w_6 & 0 \end{bmatrix},$$

where (1)  $w_j = 1/l$  when  $1 \leq j \leq l$ , (2)  $w_j = 0$  when  $l < j \leq 6$ , where  $1 \leq j, l \leq 6$ , and  $l = \left\lceil \frac{t \times 3}{50} \right\rceil$ .

After performing the ordering process to these OWA pairs according to the ordering inducing variable, we can get the ordered argument vector  $B$  shown as follows:

$$B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_6 \end{bmatrix},$$

where  $b_j$  is the  $j$ th largest of the  $DS_r(d_i)$ . The overall degree of satisfaction  $DS(d_i)$  that the document  $d_i$  satisfies the user's query can be calculated shown as follows:

$$DS(d_i) = F_w(<DS_r(d_i), DS_r(d_i)>) \\ = \sum_{j=1}^6 w_j \cdot b_j, \quad (14)$$

where  $w_j$  is the  $j$ th element of  $W$ ,  $b_j$  is the  $j$ th element of  $B$ ,  $1 \leq w_j \leq 6$ , and  $1 \leq b_j \leq 6$ .

The linguistic quantifier is easy to use since the users are not required to set the importance weights to different relevance degrees between the documents and the users' queries by different fuzzy relationships. When the users are not familiar with the definitions of the fuzzy relationships defined in the multi-relationship fuzzy concept networks, the users will be suggested to use the linguistic quantifiers. On the other hand, when the users are experienced with the multi-relationship fuzzy concept networks, they can set the importance weights to different relevance degrees between the documents and the users' queries by different fuzzy relationships directly to express their needed information more precisely. The users with little experience can give an ordering of importance weights of the degrees of satisfaction that the document satisfies the user's query by different fuzzy relationships.

## 5. CONCLUSIONS

In this paper, we have proposed a multi-relationship fuzzy concept network model. We also have presented an information retrieval method to deal with the users' fuzzy queries based on the proposed multi-relationship fuzzy concept networks. Since the concepts in the multi-relationship fuzzy concept networks are related to each other by more than one relationship at the same time, the multi-relationship fuzzy concept network model provides more representational flexibilities than the existing fuzzy information retrieval techniques. We use the IOWA aggregation operators to obtain the overall satisfaction degrees that the documents satisfy the users' queries to decide which documents should be retrieved. The users can control the aggregation processes by setting different importance weights to different fuzzy relationships in which the documents and users' queries are related. The users can set the importance weights by choosing one of the three ways provided by the proposed information retrieval method depending on their experiences. The proposed fuzzy information retrieval method is more flexible than the existing information retrieval methods.

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