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Determination of Feature Correspondences in Stereo Images Using a Calibration Polygon

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Abstract

In this paper, we propose a novel approach to solving the vertex/edge correspondence problem for stereo images. Assume an object is placed on a calibration plate (C-plate) and two perspective views of them are given. The C-plate vertex correspondence, determined by cross ratios, is used to reduce the search space for determining object vertex/edge correspondence. First, the correspondence of object edges lying on the C-plate (called the object base edges) is considered. This is because the subpolygons obtained from the division of the C-plate by the extended line of each of these base edges are viewpoint invariant that the cross ratio of each of their vertices in different images will have equal value. Assume that at least one of the base edge is visible in each image. Since a visible object base edge has to be an object boundary edge in an image, only object boundary edges will be considered for the cross ratios check for the associated subpolygons as well as other geometric constraints check in determining the object base vertex/edge correspondence. These geometric constraints include (i) the position of the edges along the boundary of object faces, and (ii) the division of the C-plate vertices by the extended lines of these edges. Based on one of the determined corresponding base edge pairs, the correspondence of all other edges can be determined by constraints similar to (i). The proposed approach only needs 2-D image data and has no constraint on the number of vertices of an object face. Experimental results are presented for some polyhedral as well as curved objects.

Keywords: Stereo vision, Feature correspondence, Calibration plate, Viewpoint invariant, Cross ratio.

1. Introduction

In stereo vision, reconstructing the 3-D information of an object requires finding feature correspondence from two perspective views of the object. Techniques for solving the correspondence problem can be divided into two categories [1, 2]: area-based stereo techniques and feature-based stereo techniques. Since area-based stereo techniques are sensitive to the change of the perspective distortions and intensity, and are slower, most researchers use the feature-based stereo techniques [3-8], which choose image features such as corner points, junctions, line segments, etc., as primitives for correspondence matching. Most feature-based stereo techniques employ epipolar line approach [3-7], such that a designated feature in one image is matched with features on the epipolar line in the other image. Since other 3-D feature points on the epipolar plane will also be projected onto the epipolar line, there are often more than one features appear on the epipolar line. In such a circumstance, one cannot be sure of the feature of the true correspondence without additional information, i.e., an ambiguity problem arises. Many researchers try to use disparities of features [3, 4], geometric constraints [5, 6], the intensity of neighborhood [7], or some other heuristics to resolve the problem, but still cannot avoid the ambiguities completely. Although the epipolar line approach searches corresponding features in a 2-D image, the approach need to use the 3-D information of two camera lens centers to construct an epipolar plane before building the epipolar line of each feature and is thus inefficient.

Lei [9] presents a method to match planar polygons of more than four vertices in two images using the cross ratios. It is shown that the value of the cross ratio of each of the vertices is viewpoint invariant. Since the method does not use 3-D information, it is simple. The method can be used to match polyhedral objects with the requirement that at least one of the polyhedral faces have more than four vertices. For every possible pair of corresponding polyhedral faces from two images, the method must compute and cyclically match the cross ratios for every vertex. Therefore, it requires much computation time and matching frequency.

In this paper, we propose a novel approach to solving the vertex/edge correspondence in the stereo images. The images are obtained with an auxiliary planar calibration plate (C-plate) placed under the object. It is shown that with the extra information provided by the C-plate, the following objectives can be attained:

(1) The number of vertices of each object face can be less than five.
(2) Only the 2-D image data is needed in determining the vertex/edge correspondence.
(3) The exhaustive search for cyclically matched cross ratio sequences for the vertices of each of the feasible object face pairs can be avoided.
(4) The ambiguity in the feature correspondence can be resolved for general situations.

According to the proposed approach, the C-plate vertex correspondence is first determined using cross ratios, the result is then used to reduce the search space for determining object vertex/edge correspondence. First, the correspondence of object edges lying on the C-plate (called the object base edges) is considered. This is because the subpolygons obtained from the division of the C-plate by the extended line of each of these base edges are viewpoint invariant that cross ratio of each of their vertices in different images will have equal value. Since a visible object base edge must be an object boundary edge in an image, only object boundary edges will be considered in determining the object base vertex/edge correspondence. The correspondence is determined by cross ratios check for the associated subpolygons as well as other geometric constraints checks about the boundary edges. These geometric constraints include (i) the position of the edges along the boundary of object faces, and (ii) the division of the C-plate vertices by the extended lines of these edges. Based on one of the determined corresponding base edge pairs, the correspondence of all other edges, including the non-base edges, can be determined by constraints similar to (i).

2. Finding Possible Base Vertex/Edge Correspondences Based on Viewpoint Invariant Features in Two Images

Assume that an object is placed on the C-plate and the boundary of the C-plate is not occluded by the object. The extended line of a base edge of the object will divide the C-plate into two subpolygons which are invariant with respect to different viewpoints. The main idea of the proposed approach is to use the viewpoint invariant property of the subpolygons to find the possible base vertex/edge correspondence.

It is obvious that any visible object base edge has to be a boundary edge of the object in an image. Thus, only these boundary edges will need to be considered in the determination of base vertex/edge correspondence. For example, Fig. 1 shows two images of a polyhedron placed on the C-plate. For the extracted object edges shown in Fig. 2, the boundary edges, represented in vertex pairs, include (0, 1), (1, 2), (2, 3), (3, 4), (4, 5), (5, 10), (10, 9) and (9, 0), while the rest are internal edges. Among these boundary edges, edges (0, 1), (1, 2), (2, 3) and (3, 4) are visible base edges. In general, the straight line containing a boundary edge can divide the C-plate into two subpolygons in the image. (See Fig. 3.) If the boundary edge is an image of a base edge, the associated subpolygons of the edge obtained from different viewpoints will correspond to the identical division of the C-plate; otherwise, the subpolygons will not have such a viewpoint invariant property.

According to Lei [9], the value of the cross ratio (which can be calculated for polygons having at least five vertices) of each of the vertices of the associated subpolygons of a base edge will be the same in images obtained from different viewpoints. Therefore, one can match the cross ratios of the vertices of the associated subpolygons of any two boundary edges, each from a different image, to determine the possible base edge correspondence. However, the direct matching is of time complexity $O(NMV)$, where N and M are the number of object boundary edges in the image 1 and image 2, respectively, and V is the maximum number of vertices of the associated subpolygons containing at least five vertices. It is shown in the following that if we first determine the vertex correspondence of the C-plates in the two images by using cross ratios, the correspondence, as an auxiliary information, can speed up the procedures for finding the base vertex/edge correspondence by reducing the search space of the matching process.

2.1. Determination of C-plate Vertex Correspondences

For determining the vertex correspondence of the C-plates in two images, it is assumed that the number of vertices of the C-plate is greater than or equal to five. Thus, the method proposed by Lei [9] can be applied. Consider the polygon shown in Fig. 4, according to Lei [9], the cross ratio of v_i is viewpoint invariant and can be computed as

$$R_1 = \frac{sin(\theta_1+\theta_2)sin(\theta_2+\theta_3)}{sin(\theta_2)sin(\theta_1+\theta_2+\theta_3)} \quad (1)$$

where θ_1, θ_2 and θ_3 are angles formed by the rays originated from V_1 and pointing toward its four neighboring vertices (V_2, V_3, V_4 and V_5) arranged in the counterclockwise (or clockwise) direction, with $\sum_{i=1}^{3}\theta_i < 180°$. Let R_j^i be the cross ratio of the *j*th vertex, V_j^i, of the C-plate in the *i*th image, $i = 1, 2; j = 1, 2, ..., V\geq 5$. For the vertex sequence $\{V_j^1 | 1 \leq j \leq V\}$, the vertex sequence $\{V_{<j+k>}^2 | 1 \leq j \leq V, 1 \leq k \leq V-1\}$, $<j+k> = (j+k) \bmod V$, whose corresponding cross ratio sequence satisfies the following criterion is identified as the matched vertex sequence:

$$\underset{k}{Min}\sum_{j=1}^{V}\left|R_j^1 - R_{<j+k>}^2\right|^2. \quad (2)$$

Thus, the computational complexity for determining the correspondence of C-plate vertices in two images is O(V), where V is the number of vertices of the C-plate. With the determined C-plate vertex correspondence, the object base vertex/edge correspondence can be found more efficiently, as discussed next.

2.2. Determination of Object Base Vertex/Edge Correspondences

Let e_i^1, $i = 1, 2, ..., N$, be the *i*th object boundary edge in image 1; and e_j^2, $j = 1, 2, ..., M$, be the *j*th object boundary edge in image 2. The procedure of determining if e_i^1 and e_j^2 form a corresponding base edge pair includes four steps described next. The first two steps check if the object faces containing these edges, as well as the divisions of C-plate by the extended lines of these edges, are geometrically consistent in the two images. The last two steps check some viewpoint invariant properties associated with the two edges.

2.2.1. Step 1 — Object Face Vertex Number Check

In this step, we check if the number of vertices of the object face containing e_i^1 is identical to that for the face containing e_j^2. Given an object boundary edge in an image, the face containing the edge is unique. Such a check can prevent two boundary edges belonging to two faces containing different number of vertices, and thus correspond to two different object edges, be processed further in the subsequent procedure.

2.2.2. Step 2 — Subpolygon's C-plate Vertex Check

It is easy to see that the division of the C-plate by the line containing e_i^1 is identical to that due to e_j^2 if the two edges correspond to the same object base edge. In this step, it is checked if the associated subpolygons of e_i^1 and e_j^2 have identical C-plate vertices whose correspondences have been found in Section 2.1.

2.2.3. Step 3 — Cross Ratio Check along Edge Direction

In Fig. 3, if edge *BC* is a base edge, an alternative form of cross ratio of the four points, *A*, *B*, *C*, *D*, on the extended line of the edge can be calculated as

$$R = \frac{|AC|.|BD|}{|BC|.|AD|}, \quad (3)$$

which is viewpoint invariant according to Duda and Hart [10]. Let the cross ratio associated with edge e_i^1 be denoted as R_i^1, and that associated with edge e_j^2 be denoted as R_j^2. We check if the two cross ratios, R_i^1 and R_j^2 are equal. The time complexity for comparing the two cross ratios is O(1). Although, for every base edge, the above cross ratios are viewpoint invariant and should have identical value in theory, numerical errors may exist in practice due to noise. (Possible sources of noise include imaging distortion, quantization error of image coordinates and feature extraction error, etc.) Table 1 lists the cross ratio values computed for the boundary edges shown in images 1 and 2 of Fig. 1 which have passed the above two steps. Two edges e_i^1 and e_j^2 are considered as a possible corresponding base edge pair, if

$$\left|R_i^1 - R_j^2\right| < T_1, \quad (4)$$

for some T_1. In this paper, the k-means clustering algorithm [11] is used to determine T_1 according to the distribution of cross ratio errors, $\left|R_i^1 - R_j^2\right|$'s, for all possible (i, j).

2.2.4. Step 4 — Cross Ratio Checks for Subpolygon's Vertices

Since the division of the C-plate by the line containing e_i^1 is identical to that due to e_j^2 if the two edges correspond to the same object base edge, the cross ratios of the corresponding vertices of the associated subpolygons of e_i^1 and e_j^2 are compared in this step to see if they possess the viewpoint invariant property. Because it is assumed that the C-plate contains at least five edges, in general, the two associated subpolygons of any one boundary edge will have four and five edges, respectively. (The discussion for the cases when a boundary edge is collinear with one or two C-plate vertices that the two associated subpolygons contain less than or equal to four edges, are omitted for brevity.) In the following calculation of cross ratios, only the associated subpolygons of e_i^1 and e_j^2 which have five edges (e.g. the subpolygons of edge *BC* which have vertices 21, 22, and 23 in Fig. 3) are considered.

Similar to the discussion presented in Section 2.1, Lei's cross ratio formula (Eq. (1)) can be used to compute the cross ratios of each vertex of the associated subpolygons of an object edge under consideration. Since the vertex correspondence of the two associated subpolygons is known from step 2, it is not necessary to cyclically match the calculated cross ratio sequences as in Eq. (2). Instead, the two vertex cross ratio sequences R^1 and R^2 ($R^k = \{R_q^k | k = 1,2; q = 1,2,\ldots,U\}$) are compared to see if the sum of the squared cross ratio differences (SCRD's) satisfies

$$\sum_{q=1}^{U} \left| R_q^1 - R_q^2 \right|^2 < T_2 \qquad (5)$$

for some small T_2, where R_q^k is the cross ratio of the *q*th vertex of the associated subpolygon in image *k*, and *U* is the total number of vertices of the subpolygon. In the presented experimental results, T_2 is heuristically chosen as three times the mean value of the SCRD's which have identical location of the most significant digit as that of the smallest SCRD. The time complexity of matching the two cross ratio sequences of the two associated subpolygons is O(1). If Eq.(5) is not true, the proposed approach precludes the possibility that the two edges form a corresponding base edge pair.

Table 2 shows possible base vertex/edge correspondences in the two images shown in Fig. 1, in which a pair of vertices is used to represent each of the object edges. For each pair of edges, one from each image, the number of steps passed in the above four-step procedure for determining possible object base edge correspondence is indicated. For example, a "3" means the two edges pass all steps except step 4. The pairs of edges marked with a "4" correspond to most likely base edge correspondences.

Although the cross ratio computed from the Lei's formula gives some reasonable results in the above example, Eq. (1) is sensitive to the variation of θ_i's in Fig. 3 for a small θ_2. The reason can be seen by observing the effect of the errors in θ_1, θ_2 and θ_3 on the error of cross ratio *R*, respectively. Let δR, $\delta\theta_1$, $\delta\theta_2$ and $\delta\theta_3$ be the errors of *R*, θ_1, θ_2 and θ_3, respectively. We have

$$\delta R = \frac{\partial R}{\partial \theta_1}\delta\theta_1 + \frac{\partial R}{\partial \theta_2}\delta\theta_2 + \frac{\partial R}{\partial \theta_3}\delta\theta_3$$

where

$$\frac{\partial R}{\partial \theta_1} = \frac{[\cos(\theta_1+\theta_2)\sin(\theta_1+\theta_2+\theta_3) - \cos(\theta_1+\theta_2+\theta_3)\sin(\theta_1+\theta_2)]\sin(\theta_2+\theta_3)}{\sin(\theta_2)\sin^2(\theta_1+\theta_2+\theta_3)},$$

$$\frac{\partial R}{\partial \theta_2} = \frac{1}{\sin^2(\theta_2)\sin^2(\theta_1+\theta_2+\theta_3)} \times$$
$$\{[\cos(\theta_1+\theta_2)\sin(\theta_2+\theta_3)+\cos(\theta_2+\theta_3)\sin(\theta_1+\theta_2)]\sin\theta_2\sin(\theta_1+\theta_2+\theta_3) -$$
$$[\cos(\theta_2)\sin(\theta_1+\theta_2+\theta_3)+\cos(\theta_1+\theta_2+\theta_3)\sin(\theta_2)]\sin(\theta_1+\theta_2)\sin(\theta_2+\theta_3)\},$$

and

$$\frac{\partial R}{\partial \theta_3} = \frac{[\cos(\theta_2+\theta_3)\sin(\theta_1+\theta_2+\theta_3) - \cos(\theta_1+\theta_2+\theta_3)\sin(\theta_2+\theta_3)]\sin(\theta_1+\theta_2)}{\sin(\theta_2)\sin^2(\theta_1+\theta_2+\theta_3)}.$$

When $0° < \theta_2 < 90°$, a small θ_2 will result in a large value of $\partial R/\partial\theta_i$ because of the $\sin(\theta_2)$ term in the denominator of $\partial R/\partial\theta_i$ in the above equations. For example, the associated subpolygon of edge (0, 1) in image 1 (and in image 2) shown in Fig. 3 has two very short edges, (51, 23) and (21, 50). Vertex 21 of the associated subpolygon of edge (0, 1) in image 1 (2) has $\theta_2 = 10.57$ (11.8) and vertex 23 in image 1 (2) has $\theta_2 = 4.8$ (7.73). Table 3 shows the cross ratios calculated for the vertices of the associated subpolygons of edge (0, 1) in the two images. Since the two cross ratios, as well as the SCRD, calculated for vertex 21 are large (similar result can be found for vertex 23), the total SCRD is equal to 1.56 and the two corresponding base edges do not pass the cross ratio check in step 4. Similar result can be shown for edge (1, 2). In fact, there are examples for which the cross ratio check yields erroneous results, i.e., the smallest total SCRD does not correspond to a correct base edge correspondence.

In order to avoid the above undesirable numerical problems due to very short edges of the associated subpolygons, different ways to compute the cross ratios are considered in this paper. Barrett et al. [12] have shown that if no three of five coplanar points are collinear, the cross ratio of one of the points can be calculated as

$$R^* = \frac{\sin\theta_2\sin\theta_4}{\sin(\theta_1+\theta_2)\sin(\theta_2+\theta_3)} \qquad (6)$$

where θ_1, θ_2 and θ_3 are defined similar to that shown in Fig. 4 and $\theta_4 = 360° - \sum_{i=1}^{3}\theta_i$. Since Eq. (6) is the negative reciprocal of Eq.(1), the cross ratio R^* also possess the viewpoint invariant property. Since Eq. (6) does not have the aforementioned numerical problem for a small θ_2, it is used in the rest of this paper for the cross ratio check in step 4. Table 4 shows the results similar to that shown in Table 2 by using (6) in place of (1). Each of the boundary edge pairs passing all four steps corresponds to a correct base edge correspondence. Among these corresponding base edge pairs, the one with the minimum total SCRD will be used to determine the correspondence of all other object edges, as discussed next.

3. Determination of Other Vertex/Edge Correspondences

Once the corresponding object base edge pair is found with the above procedure, the result is used to find the correspondences of other edges in the two images. Since each of the pair of the corresponding base edges belongs to only one object face in the individual image, the two faces can be matched using their edge sequences arranged in the clockwise (or counterclockwise) order. The matching of the two faces is considered successful if the following two conditions hold: (a) the lengths of the two edge sequences are equal and (b) the newly matched pairs of edges do not conflict with any existing matched edge pairs. After a successful matching of two faces, the two faces are marked "matched" and all the newly matched pairs of edges are recorded and attached to the end of the "matched edge pair (MEP)" queue. The above procedure continues for the next matched edge pair in the MEP queue to check whether the two edges belong to two object faces which satisfy the above two conditions but have not yet been matched. The process for determining the vertex/edge correspondence is completed when the MEP queue is empty.

For the example shown in Fig. 1, the correspondences of all the object edges in the two images can be found correctly with the above procedure. The proposed procedure also works well for partially occluded object (examples are presented later in Section 5). However, there are rare occasions which occur at specific viewpoints for some special object shapes that additional procedures are required to ensure meaningful results in the correspondence. The procedures are omitted for brevity.

4. Experimental Results

In this section, experimental results for the determination of vertex/edge correspondences for an object in two images are presented. For an object placed on the C-plate, two images are taken from two different viewing angles, denoted as Image 1 and Image 2 of the object, respectively. Fig. 6 shows the 512×512 images of the polyhedron with edges of the polyhedron marked with dark lines. Once images of an object are obtained, line features are extracted for the object as well as the C-plate. The intermediate experimental results for Fig. 6 are not shown for brevity. Fig. 7 shows the corresponding edges obtained by the proposed approach for Fig. 6. The proposed approach also works for curved objects. Fig. 8 shows the images of the curved object with dark grid lines are pasted on its surface, and with endpoints of selected object edges marked with identification numbers. Table 5 shows the results of the base edge correspondence finding procedure for selected edges shown in Fig. 8 for the curved object. One can verify that the base edge correspondence for the curved object is correct. The other corresponding edges similar to that shown in Fig. 7 can also be derived for the curved object. The results are not shown for brevity.

5. Conclusion

In this paper, we have presented a novel approach which uses the extra information provided by a calibration polygon, the C-plate, to solve the object vertex/edge correspondence problem. We first determine the C-plate vertex correspondence in two images by comparing the cross ratios of C-plate vertices. The result is then used to simplify the subsequent procedure for finding the object base vertex/edge correspondence. Only object boundary edges in the two images need to be considered for possible base edge correspondence. Simple geometric relations with other features in the image as well as viewpoint invariant properties associated with a base edge are examined in finding the correspondence. The former are concerned with (i) the object face containing the edge and (ii) the subpolygons obtained by dividing the C-plate with the extended line of the edge. The latter, on the other hand, involve the calculation of the viewpoint invariant measure, e.g., cross ratio, for (i) relative location of the edge in the C-plate along the direction of the edge and (ii) the associated subpolygons. Once the object base edge correspondence has been determined, the correspondence of other edges can be determined in an straightforward manner using one of the corresponding base edge pairs, except for some very special situations. The special situations are involved with images acquired at specific viewpoints for some special object shapes, and may result in ambiguity in the correspondence. Additional procedure may be carried out to resolve the problem.

The proposed approach has several advantages compared with other approaches for solving similar problems. First, it uses only 2-D image data and does not require 3-D information, so it is simple and efficient. Second, unlike Lei's method, the number of vertices of an object face can be less than five. Third, with the predetermined vertex correspondence of the C-plate, the search space for finding base vertex/edge correspondence is greatly reduced. Finally, the ambiguity in the correspondence can be resolved for general situations.

The proposed approach can be extended to situations when the C-plate boundary is partially occluded as long as the complete boundary can be recovered.

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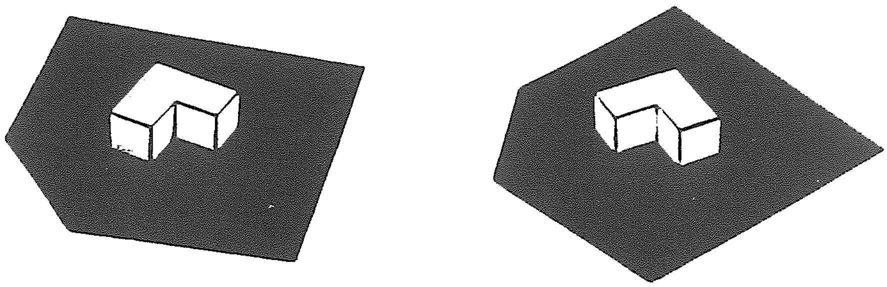
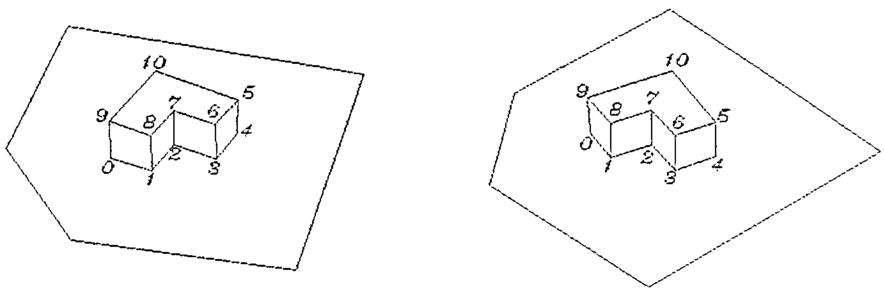


(a) (b)

Fig. 1. Two views of a polyhedron placed on the C-plate. (a) Image 1. (b) Image 2.



(a) (b)

Fig. 2. Edges extracted from (a) Image 1 and (b) Image 2 of Fig. 1.

Table 1. Cross ratios of the four points, similar to that shown in Fig. 3, along each of the extended line of the boundary edges shown in Fig. 2 which pass the two steps of the procedure for finding the object base edge correspondence.

Image 1		Image 2	
Vertex pairs of boundary edges	Cross ratios R_i^1	Vertex pairs of boundary edges	Cross ratios R_j^2
(0, 1)	2.24	(0, 1)	2.27
(1, 2)	2.28	(1, 2)	2.30
(2, 3)	2.20	(2, 3)	2.24
(3, 4)	2.35	(3, 4)	2.34
(4, 5)	1.83	(5, 10)	1.40
(5, 10)	1.42	(10, 9)	1.26
(10, 9)	1.27		

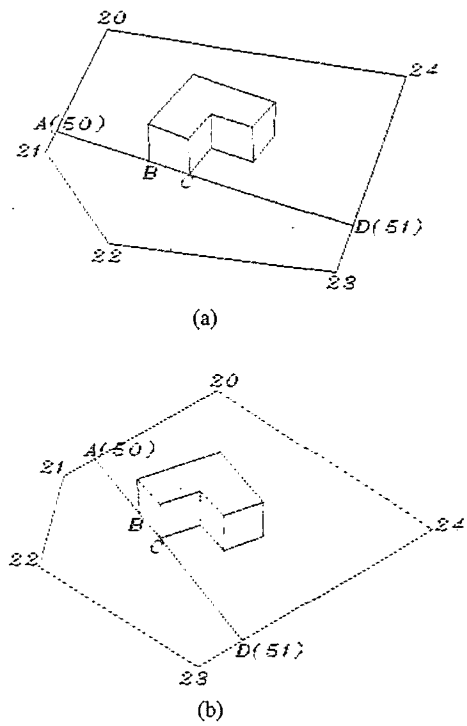


Fig. 3. The division of the C-plate into two associated subpolygons by the extended line of the object boundary edge BC in (a) Image 1 and (b) Image 2.

Table 2. The number of steps passed in the four-step procedure for finding the object base edge correspondence for boundary edges shown in Fig. 2.*

Total steps passed / Boundary edge in image 2 / Boundary edge in image 1	(0, 1)	(1, 2)	(2, 3)	(3, 4)	(4, 5)	(5, 10)	(10, 9)	(9, 0)
(0, 1)	3	1	2	1	1			1
(1, 2)	1	3	1	3	1			1
(2, 3)	2	1	4	1	1			1
(3, 4)	1	3	1	4	1			1
(4, 5)	1	2	1	2	1			1
(5, 10)						4	1	
(10, 9)						1	3	
(9, 0)	1	1	1	1	1			1

* In step 4, the cross ratios are computed using Lei's cross ratio formula (Eq. (1)).

Table 3. Cross ratios* of vertices of the pentagonal subpolygon of edge (0, 1) shown in Image 1 and Image 2 of Fig. 2, and the sum of the squared differences of the corresponding cross ratios.

Edge (0, 1) in image 1		Edge (0, 1) in image 2		Absolute difference of cross ratios $\|R_q^1-R_q^2\|$	$\|R_q^1-R_q^2\|^2$
Vertex # of an associated subpolygon	Cross ratio R_q^1	Vertex # of an associated subpolygon	Cross ratio R_q^2		
50	1.21	50	1.16	0.05	0.00
51	1.22	51	1.28	0.06	0.00
23	4.56	23	3.91	0.66	0.43
22	1.04	22	1.05	0.00	0.00
21	4.73	21	5.79	1.06	1.13
Sum					1.56

* The cross ratios are computed using Lei's cross ratio formula.

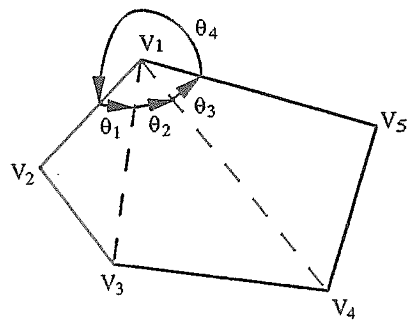
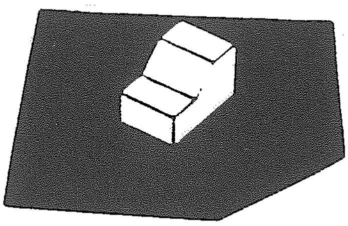


Fig. 4. Internal/external angles of a pentagon formed by $\overrightarrow{V_1V_i}$'s.

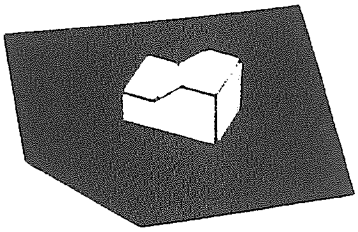
Table 4. The number of steps passed in the four-step procedure for finding the object base edge correspondence for boundary edges shown in Fig. 2.*

Total steps passed / Boundary edge in image 2 (cols) / Boundary edge in image 1 (rows)	(0, 1)	(1, 2)	(2, 3)	(3, 4)	(4, 5)	(5, 10)	(10, 9)	(9, 0)
(0, 1)	3	1	2	1	1			1
(1, 2)	1	4	1	3	1			1
(2, 3)	2	1	4	1	1			1
(3, 4)	1	3	1	4	1			1
(4, 5)	1	2	1	2	1			1
(5, 10)						3	1	
(10, 9)						1	3	
(9, 0)	1	1	1	1	1			1

* In step 4, the cross ratios are computed using Barrett's cross ratio formula (Eq. (6)).

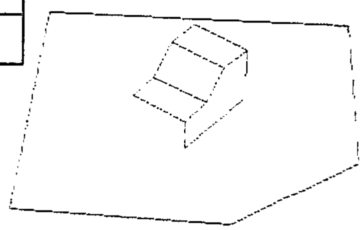


(a)

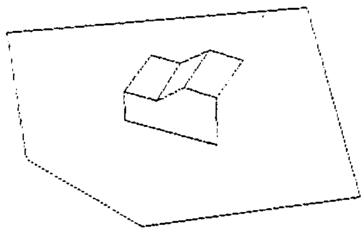


(b)

Fig. 5. Two views of a polyhedron placed on the C-plate. (a) Image 1. (b) Image 2.

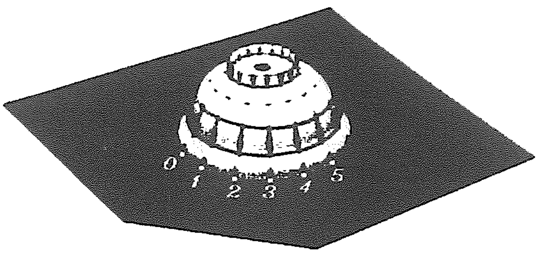


(a)

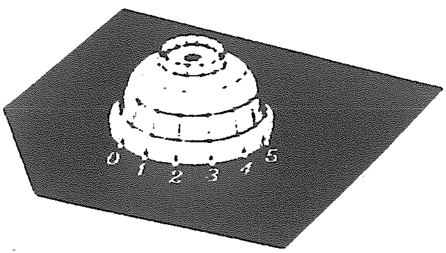


(b)

Fig. 6. Edges in (a) Image 1 and (b) Image 2 of Fig. 5 for which the vertex/edge correspondences have been established.



(a)



(b)

Fig. 7. The two views of the bowl-shaped object placed on the C-plate with identification numbers marked for some vertices. (a) Image 1. (b) Image 2.

Table 5. The number of steps passed in the four-step procedure for finding the object base edge correspondence for some selected boundary edges shown in Fig. 7.

Total steps passed / Boundary edge in image 2 (cols) / Boundary edge in image 1 (rows)	(0, 1)	(1, 2)	(2, 3)	(3, 4)	(4, 5)
(0, 1)	1	1	1	1	1
(1, 2)	1	1	1	1	1
(2, 3)	1	1	1	1	1
(3, 4)	2	1	1	1	1
(4, 5)	1	4	2	1	1