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Resolution Enhancement using EM Algorithm

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ABSTRACT

The EM (Expectation-Maximization) algorithm is a broadly applicable method for calculating maximum likelihood estimates given incomplete data[1]. The EM algorithm has received considerable attention due to their computation feasibility in tomographic image reconstruction[2~5], and parameter estimation[6]. However, it is less recognized that the EM algorithm can be equally applicable to image enhancement applications encountered in scanning, reproduction and rendering processes. No past techniques surveyed can incorporate the potentially complex nature of various image formation processes into a simple probability density array as the EM procedure does. In this paper, an image enhancement technique utilizing the EM procedure to model the image formation process is proposed. By dynamically giving *a priori* probability distribution suited for a specific application environment currently considered, the proposed method provides a general framework for rendering good image quality at the designated resolution for a large class of image formation processes.

1. INTRODUCTION

The Expectation-Maximization (EM) algorithm was first used for calculating maximum likelihood (ML) estimations given incomplete data[1]. The EM algorithm incorporates the potentially complex nature of image formation process into a simple probability array and uses iterative method to restore the complex nature of image formation process. Hence, in recent years, the EM paradigm has been applied to tomographic image reconstruction [2], symbol detection[7], parameter estimation[8], and time delay estimation for filtered Poisson Processing[6]. However, no past techniques surveyed can incorporate the potentially complex nature of various image formation processes into a simple probability density array as the EM procedure does. In this paper, a resolution and contrast enhancement method utilizing the EM procedure is proposed.

EM algorithm

The EM algorithm is a very general iterative algorithm for ML estimation in incomplete data problems. In fact, the range of problems that can be attacked by the EM is very broad and includes applications not usually considered to be ones arising from missing or incomplete data, e.g., variance components estimation, iteratively reweighted least squares[11]. The EM algorithm formalizes a relatively old ad hoc idea for handling incomplete data by replacing incomplete values by estimated values first, then estimate parameters. The newly acquired parameters is used to reestimated the incomplete values assuming the new parameter estimates are correct. The whole procedure is repeated until parameter converges.

The EM algorithm is modeled as the complete data log-likelihood function

$$l(X|\theta) = \ln L(X|\theta),$$

where X is the complete data, l is the function of the log-likelihood function, L is the likelihood function, θ is the parameter needed to be estimated. More generally, missing sufficient statistics rather than individual observations need to be estimated, and even more generally, the log-likelihood $l(X|\theta)$ itself needs to be estimated at each iteration of the algorithm.

The earliest reference surveyed comes from McKendrick, who applies the EM algorithm to a medical application[12]. Hartley develops the theory quite extensively and considers the general case of counted data[11]. Baum *et al.* uses the algorithm in a Markov model and proves essential mathematical results, results that can be easily generalized[15]. Orchard and Woodbury first notice the general applicability of the underlying idea, calling it the "missing information principle[13]." Sundberg explicitly considers properties of the general likelihood equations[14], and Beale and Little further develop the theory for the normal model[10]. The term EM is introduced in Dempster, Laird, and Rubin[1], and their work exposed the full generality of the algorithm by (1) proving general results about its behavior, specifically that each iteration increases the log-likelihood $l(Y|\theta)$, where Y is the incomplete data, and (2) providing a wide range of examples. Since 1977, there have been many new usages of the EM algorithm, as well as works relating to its convergence properties[9].

Each iteration of the EM consists of an E step (expectation step) and an M step (maximization step). These steps are often easy to construct conceptually, to program for calculation, and to fit into computer storage. Also each step has a direct statistical interpretation. An additional advantage of the algorithm is that it can be shown to converge reliably, in the sense that under general conditions, as the number of iteration increases, the log-likelihood $l(Y|\theta)$ converges [1]. The EM algorithm has the following desirable properties: (1) If $l(X|\theta)$ is an exponential family and $l(Y|\theta)$ is bounded, then $\theta^{(t)}$, the t^{th} iteration of parameter θ, converges. (2) If $l(Y|\theta)$ is bounded then $l(Y|\theta^{(t)})$ will converge. (3) Monotonic increase of $l(Y|\theta)$ at each iteration to a unique point to ensure the validity of the algorithm. A disadvantage common to the EM algorithm is that its rate of convergence can be painfully slow if a lot of data are unobserved.

The E step and M step

The E (expectation) step finds the conditional expectation of the "complete data" given the observed data and current estimated parameters. Specifically, let $\theta^{(t)}$ be the current (t^{th}) estimate of the parameter θ. In the E-step, a conditional expectation is formulated by defining:

$$Q(\theta|\theta^{(t)}) \equiv E(l(X|\theta) \mid Y, \theta^{(t)}),$$

where X represents the complete data set, Y the incomplete data set, and $E(.)$ the mathematical expectation value.

The M (Maximization) step is simple to describe: perform maximum likelihood estimation of θ. In fact, the M-step obtains a set of new estimates by maximizing the conditional expectation formulated with respect to the parameters obtained in the E-step. Effectively, the M step of the EM algorithm uses the same computational methods as ML estimation from $l(X|\theta)$. The M step of the EM determines $\theta^{(t+1)}$ by maximizing the expected log-likelihood.

Starting the EM algorithm by arbitrary assigning the initial value of the parameter needs to be estimated. Then from the E-step, a new formula is generated by deriving the conditional expectation with respect to the parameter, followed by the M-step, a set of new estimate with respect to the parameter is formed by solving the equation generated from E-step. By following this procedure, an iteration of the EM algorithm is complete. This iterative process has the desirable properties of maximizing the likelihood function defined on the measured data monotonically[1], and converging to a global maximum at a unique point[9]. The E-step followed by the M-step is computed iteratively until the parameters converged. Since each iteration of the algorithm consists of an expectation step followed by a maximization step, this iterative procedure is called the EM algorithm. The EM algorithm is useful because of its computational simplicity, and provides a numerical method to find the ML estimates.

A multinomial example[1]

Suppose that the data vector of observed counts $Y = (38, 34, 125)$ is postulated to arise from a multinomial with cell probabilities

$$\left(\frac{1}{2}-\frac{1}{2}\theta, \frac{1}{4}\theta, \frac{1}{2}+\frac{1}{4}\theta\right).$$

The objective is to find the ML estimate of θ. Define $X = (y_1, y_2, y_3, y_4)$ to be multinomial with probability

$$\left(\frac{1}{2}-\frac{1}{2}\theta, \frac{1}{4}\theta, \frac{1}{4}\theta, \frac{1}{2}\right).$$

where $Y = (y_1, y_2, y_3+y_4)$. Notice that if Y were observed, the ML estimate of θ would be immediate:

$$\frac{y_2 + y_3}{y_1 + y_2 + y_3}.$$

Also note that the log-likelihood $l(X|\theta)$ is linear in X, so finding the expectation of $l(X|\theta)$ given θ and Y involves the same calculation as finding the expectation of X given θ and Y, which in effect fills in estimates of the incomplete values:

$$E(y_1|\theta, Y) = 38,$$

$$E(y_2|\theta, Y) = 34,$$

$$E(y_3|\theta, Y) = 125\frac{1}{4}\theta \Big/ \left(\frac{1}{2}+\frac{1}{4}\theta\right),$$

$$E(y_4|\theta, Y) = 125\left(\frac{1}{2}\right) \Big/ \left(\frac{1}{2}+\frac{1}{4}\theta\right).$$

Thus at the t^{th} iteration, with estimate , we have for the E step

$$y_3^{(t)} = 125\left(\frac{1}{4}\theta^{(t)}\right) \Big/ \left(\frac{1}{2}+\frac{1}{4}\theta^{(t)}\right),$$

and for the M step, we have

$$\theta^{(t+1)} = \left(34 + y_3^{(t)}\right) \Big/ \left(72 + y_3^{(t)}\right).$$

Iteration between the E step and M step defines the EM algorithm for this problem. In fact, setting $\theta^{(t+1)} = \theta^{(t)} = \hat{\theta}$ and combining these two equations yields a quadratic in $\hat{\theta}$ and thus a closed-form solution for the ML estimate.

Our work

A schematic representation of the application of the EM algorithm to resolution enhancement encountered in the scanning process is shown in Figure 1. A fluorescent light source pulled by a tracking mechanism from top to bottom emits light on the surface of the scanned hardcopy. The intensity of the light reflected is then detected by an array of light sensors. However, the intensity registered by any specific sensor is affected not only by the area bounded by scanner resolution, but also neighboring areas due to the diffusion of light. The proposed method utilizing the EM procedure can be used to compensate the above effect, restore original density, or increase resolution if desired.

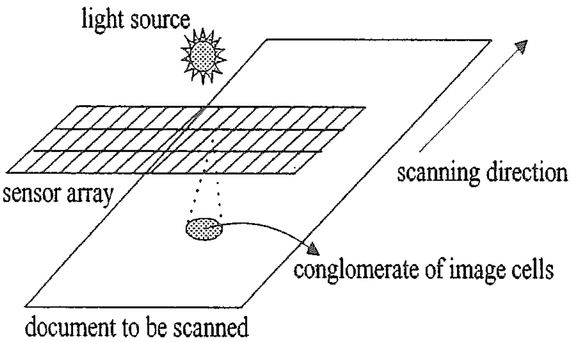

Figure 1 Image formation process and the spatial relationship between sensor array and image cells.

Let the intensities detected by the sensor array correspond to the incomplete data set $\{Y_{ij}\}$, and the complete data set $\{X_{ij,mn}\}$, defined as the physically un-measurable actual image intensity located at cell (m, n), distributed by parameter yet to be determined λ_{mn}, and registered in sensor (i, j). The dimensions of Y and λ need not to be the same. Due to the hardware cost, the number of sensors is almost always less than the number of image cells in the actual application. Hence, the problems we are targeting can be stated as: "*Given the limited spatial resolution of sensor array, how can we best compensate the physical configuration of the underlying imaging digitization system and render the output at the desired resolution?*" No past techniques surveyed can incorporate the potentially complex nature of image formation process into a simple probability array as the EM procedure does. Factors affecting the sampling processing include type of light source, the spatial configuration between light source, the scanned document and sensor array, the surface of the origin which affects the pattern of light scattering, the sensitivity of sensors and registration errors, etc. The new enhancement method proposed in this paper can obtain resolution at any desired scale given the intensities registered by the finite number of sensors, and the spatial correspondence between sensors (i, j) and image cells (m, n). The EM operation for a one-to-one mapping between X and Y represents image density compensation or re-calibration, while a many-to-one mapping means resolution enhancement. By incorporating all physical factors involved in the image formation process, the probability density function $\{C_{ij,kl}\}$ representing the light intensity emitted from cell (m, n) and detected by sensor (i, j) can be formed. An iterative equation formulated by employing the EM paradigm is as follows:

$$\lambda_{mn}^{(t+1)} = \left(\frac{\lambda_{mn}^{(t)}}{\sum_i \sum_j C_{ij,mn}} \right) \left(\sum_i \sum_j C_{ij,mn} \frac{Y_{ij}}{\sum_r \sum_s C_{ij,rs} \lambda_{rs}^{(t)}} \right)$$

Where $\lambda_{mn}^{(t)}$ represents the t^{th} estimate of λ_{mn}.

The same principle can be adapted to various imaging systems by incorporating all physical factors into $\{C_{ij,mn}\}$ and specifying different spatial relationship between cells and sensors. Take a rod-like arrangement of sensors for example, the $\{C_{ij,mn}\}$ can be confined to a rectangular area, served as an indication for the line sensor setup, with amplitude inverse proportional to the distance from the center cell considered, reflecting an energy drop (ref. Figure 2). If a one-to-one spatial mapping between image cell and sensor is specified, then the EM procedure redistributes the intensity of image cell detected according to the probability density matrix $\{C_{ij,mn}\}$ postulated. On the other hand, if a many-to-one mapping is specified, an interpolation of cells is performed based on the cells detected and the probability density matrix $\{C_{ij,mn}\}$. An enhancement in resolution is achieved. In comparison with other interpolation techniques, e.g., linear interpolation between neighboring image cells, the EM algorithms has the advantage of incorporating all the physical factors affecting a complex image formation into a single probability density matrix $\{C_{ij,mn}\}$ and thus gives better results.

0.00030	0.00174	0.01373	0.03662	0.06409	0.07690	0.06409	0.03662	0.01373	0.00174	0.00030
0.00036	0.00367	0.01648	0.04394	0.07960	0.09229	0.07960	0.04394	0.01648	0.00367	0.00036
0.00030	0.00174	0.01373	0.03662	0.06409	0.07690	0.06409	0.03662	0.01373	0.00174	0.00030

center cell

Figure 2 The probability density matrix $\{C_{ij,mn}\}$ for an exemplary rod-like arrangement of sensors.

The EM algorithm can use not only in resolution enhancement but also in image scaling operations. In the past, methods used for changing the scale of an image are usually simple duplication or linear interpolation. In simple duplication technique, a pixel is filled with duplicated luminance value of existing nearest neighboring pixel. The linear interpolation method uses linear weighting for assigning new pixel values according to distances to existing neighboring pixels. The interpolated pixel value is a linear function of the distance to its neighboring sampled pixels. This method usually has better interpolated result than the simple duplication one. However, both methods tend to blur the image and reduce the high frequency component. In contrast, the EM algorithm uses probability density function to represent the complex nature of image. It may include all the configuration factors that may affect the image scaling. Thus, it has the potential to produce better image quality than those of simple duplication method and interpolation method. The EM algorithm is an image sharpening process that can restore the fine details of a blurred image.

2. THE IMAGE MODEL AND FORMULA DERIVATION

In this section, we present a contrast and resolution enhancement technique by incorporating the potentially complex nature of various image formation processes into a simple probability density array using EM algorithm. No past techniques surveyed can represent such complex nature of various image formation process into a simple probability density array. Restoration of the original intensity or increasing the image resolution can be reached. The spatial arrangement of photodetecting array, suited for a specific application environment, is used as *a priori* information by setting up a corresponding probability distribution. Through the EM iterative formulation, the restored image intensity is obtained at the desirable preset resolution. Contrast enhancement using a Gaussian blurred image as test image

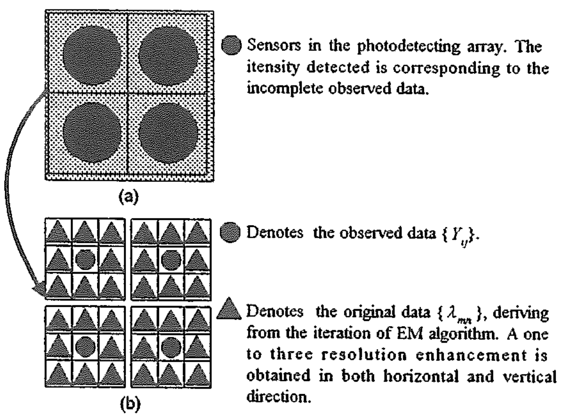

Figure 3 The image model. (a) The observed image, incomplete data set in the EM formulation, from the scanning process. (b) The image after a scale change of one to three.

is performed. The result obtained from enlarging an image using the EM algorithm is compared with those by simple duplication and linear interpolation methods. The simulation study shows the EM algorithm is a feasible method for a large class of image processing applications.

The Image Model

The relationship between observed data and parameters to be estimated in using the EM formulation is described first. Consider the diagram shown in Figure 3. Figure 3(a) represents the pixels obtained from an image scanning device. It is an array of grayscale image with size *N⊠M*. Let i and j represent the horizontal and vertical coordinates on the plane of the sensor array respectively, where $i = 1,\ldots,N$ and $j = 1,\ldots,M$. The gray level of the observed image detected by the $(i, j)^{th}$ sensor on the photodetecting array is denoted as Y_{ij}. Figure 3(b) represents an image after performing scale change of the original scanned image in (a). A one to three scale change is demonstrated in both the horizontal and vertical directions. Therefore, in our EM formulation, Y_{ij}, the pixel value obtained from the scanning process, corresponds to the incomplete data, while the newly added pixel value λ_{mn} maps to the parameters yet to be determined. The complete data set $\{X_{ij,mn}\}$ represents the number of photons emitting from cell (m, n) and detected by sensor located in (i, j). This complete data set can not be measured physically. Given the observed, incomplete data set $\{Y_{ij}\}$, which is limited by the resolution of sensor array, and the specified mapping relationship between $\{Y_{ij}\}$ and an unobservable complete data set $\{X_{ij,mn}\}$, which corresponds to the image plane resolution, the EM algorithm can estimate the parameters iteratively to restore the correct value of unobservablecomplete data set $\{X_{ij,mn}\}$.

In summary, the following symbols are used in the formulation of the EM algorithm:

1. $\{Y_{ij}\}$ is the luminance value detected by the sensor located in (i, j). It also corresponds to the observable, incomplete data in our EM formulation.
2. $\{X_{ij,mn}\}$ is the number of photons emitted from cell (m, n) and detected by sensor located in (i, j), This corresponds to the unobservable, complete data in our EM formulation. $X_{ij,mn}$ can not be measured physically. Normally, the observable data Y_{ij} is an accumulation of the photons emitted from all the image cells and detected by sensor located in (i, j), i.e., $Y_{ij} = \sum_m \sum_n X_{ij,mn}$.
3. $\{\lambda_{mn}\}$ is the total number of photons emitted from image cell (m, n). The photons emitted may be detected by more than one sensors due to the angle of emission, i.e., $\lambda_{mn} = \sum_i \sum_j X_{ij,mn}$ .This is the set of parameters needed to be estimated.
4. Given observed, fixed resolution $i \times j$ image, we can stipulate the dimensions of m and n, $m \ge i, n \ge j$,

the mapping between (m, n) and (i, j) according to certain image formation model, and formulate an EM iterative procedure to obtain an image with better resolution $m \times n$.

Formula Derivation

According to the notations given above, we can establish the following relationship:

$$Y_{ij} = \sum_m \sum_n X_{ij,mn} \quad (1)$$

With the photon emission behavior, the expected value with the configuration of the photon emission, the expected value of $X_{ij,mn}$ given λ, can be defined as:

$$N_{ij,mn} \equiv E(X_{ij,mn} | \lambda) = C_{ij,mn} \lambda_{mn} \quad (2)$$

where $\{C_{ij,mn}\}$ are the probability that photons emitted from cell (m, n) and detected by sensor (i, j). In principle, $\{C_{ij,mn}\}$ can incorporate every physical factor involved in the image formation process, not just the geormetric orientation between image cell and sensor location mentioned previously. Thus, it can be assigned dynamically to meet the requirement of specific imaging system. From the Poisson nature of the photon emission process, the conditional probability density function of $\{X_{ij,mn}\}$ given λ is:

$$P(X_{ij,mn} | \lambda) = \frac{N_{ij,mn}^{X_{ij,mn}}}{X_{ij,mn}!} \exp(-N_{ij,mn}).$$

All the elements of $\{X_{ij,mn}\}$ are independently identically distribution (*iid*). The likelihood function of the complete data set $L(X|\lambda)$, the likelihood function by definition, can be expressed in terms of the joint probability density function as:

$$L(X | \lambda) = \prod_i \prod_j \prod_m \prod_n P(X_{ij,mn} | \lambda).$$

Substituting the expression for $P(X_{ij,mn}|\lambda)$ back to the above equation, we can rewrite the likelihood function as:

$$L(X | \lambda) = \prod_i \prod_j \prod_m \prod_n \frac{N_{ij,mn}^{X_{ij,mn}}}{X_{ij,mn}!} \exp(-N_{ij,mn}).$$

Take the logarithm of the above likelihood function on both sides, we obtain

$$\ln[L(X | \lambda)] = \ln\left[\prod_i \prod_j \prod_m \prod_n \frac{N_{ij,mn}^{X_{ij,mn}}}{X_{ij,mn}!} \exp(-N_{ij,mn})\right]$$

$$= \sum_i \sum_j \sum_m \sum_n \left[-N_{ij,mn} + X_{ij,mn} \ln(N_{ij,mn}) - \ln(X_{ij,mn}!)\right]$$

From the above equation, we can then proceed with the E-step and M-step of the EM algorithm.

The E-step: In the E-step of the EM algorithm, a conditional expectation of $\ln[L(X|\lambda)]$ with respect to the incomplete data set, $\{Y_{ij}\}$, and the current(t^{th}) estimates, $\lambda^{(t)}$, is formulated by defining:

$$Q_E(\lambda | \lambda^{(t)}) \equiv E\left[\ln L(X | \lambda) | Y, \lambda^{(t)}\right] \quad (3)$$

$$= \sum_i \sum_j \sum_m \sum_n \left[-N_{ij,mn} + E(X_{ij,mn} | Y_{ij}, \lambda^{(t)}) \cdot \ln N_{ij,mn}\right] + Z$$

where Z is independent of λ. By the definition of conditional expectation and the relationship between $\{Y_{ij}\}$ and $\{X_{ij,mn}\}$ from equation (1), the following equation can be derived.

$$E(X_{ij,mn} | Y_{ij}, \lambda^{(t)}) = E\left(X_{ij,mn} \Big| \sum_m \sum_n X_{ij,mn}, \lambda^{(t)}\right) \quad (4)$$

$$= \frac{\left(\sum_m \sum_n X_{ij,mn}\right) E(X_{ij,mn} | \lambda^{(t)})}{\sum_r \sum_s E(X_{ij,rs} | \lambda^{(t)})} = \frac{Y_{ij} E(X_{ij,mn} | \lambda^{(t)})}{\sum_r \sum_s E(X_{ij,rs} | \lambda^{(t)})}$$

According to the definition of the expectation value of $X_{ij,mn}$ given λ from equation (2), replace λ with the current (t^{th}) estimation $\lambda^{(t)}$, and λ_{mn} by and its $\lambda_{mn}^{(t)}$.

$$E(X_{ij,mn} | \lambda^{(t)}) = N_{ij,mn}^{n} = C_{ij,mn} \lambda_{mn}^{(t)}. \quad (5)$$

Substituting equations (4) and (5) into (3), we can obtain the following equation:

$$Q_E(\lambda | \lambda^{(t)}) =$$

$$\sum_i \sum_j \sum_m \sum_n \left[-C_{ij,mn} \lambda_{mn} + \left(\frac{C_{ij,mn} \lambda_{mn}^{(t)} Y_{ij}}{\sum_r \sum_s C_{ij,rs} \lambda_{rs}^{(t)}}\right) \ln(C_{ij,mn} \lambda_{mn})\right] + Z$$

The M-step: The M-step is fairly straightforward, the next $(t+1)^{th}$ estimation of λ is calculated by maximizing $Q_E(\lambda|\lambda^{(t)})$ with respect to λ_{mn}. First, differentiate $Q_E(\lambda|\lambda^{(t)})$ with respect to λ_{mn}, and let it equal to zero.

$$\left(\frac{\partial}{\partial \lambda_{mn}}\right) Q_E(\lambda | \lambda^{(t)}) = \sum_i \sum_j \left[-C_{ij,mn} + \frac{C_{ij,mn} \lambda_{mn}^{(t)} Y_{ij}}{\lambda_{mn}\left(\sum_r \sum_s C_{ij,rs} \lambda_{rs}^{(t)}\right)}\right] = 0.$$

The above equation is solved to yield the next $(t+1)^{th}$ estimate of λ:

$$\lambda_{mn}^{(t+1)} = \left(\frac{\lambda_{mn}^{(t)}}{\sum_i \sum_j C_{ij,mn}}\right) \left(\sum_i \sum_j C_{ij,mn} \frac{Y_{ij}}{\sum_r \sum_s C_{ij,rs} \lambda_{rs}^{(t)}}\right).$$

The E-step and M-step are repeated iteratively until $\{\lambda_{mn}\}$ converges. The probability density of complete data set $\{X_{ij,mn}\}$ and incomplete data set $\{Y_{ij}\}$ are exponential family. Hence, from the property of the EM algoirthm, through the iterative method, the EM algorithm guarantees the parameter will converge to an unique global maximum point.

3. SIMULATION RESULTS

The EM procedure developed is applied to the contrast and resolution enhancement of test images in this section. The simulation is divided into two parts. In the first part, the original *256×256* Lena image is blurred by using a mask following Gaussian amplitude, simulating the light scattering pattern occurred in the scanning process. The dimensions of the image before and after applying the EM procedure are the same. The image is deblurred by determining the $\{C_{ij,mn}\}$ first, then following the expectation and maximization steps iteratively. In the second part, The original Lena image is scale down first, then enlarged it in a one-to-three scale horizontally and vertically by the EM algorithm, simple duplication method and linear interpolation method.

Deblurring simulation

The image is first blurred by using a mask with Gaussian amplitude. The values of the mask are determined according to the Gaussian distribution. That is, the value of the element in the mask depends on the distance between the element and the center element. The Gaussian distribution has the probability density function as following:

$$P(x) \sim G(\mu, \sigma^2),$$

$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

where μ is the mean value of the samples, σ is the variance of the samples in the probability sense. The value of the μ and σ can be specified directly. The element within the mask is evaluated according to the following formulation:

$$m(x,y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-c_x)^2+(y-c_y)^2}{2\sigma^2}},$$

where, c_x is the x position of center element and c_y is the y position of center element. The center element is the element under consideration. The gray value of the pixel is then determined by the following formula:

$$NewGrayValue(x,y) = \sum_{i=x-3}^{x+3} \sum_{j=y-3}^{y+3} m(i,j) * OrgGrayValue(i,j)$$

where *NewGrayValue*(*x*, *y*) is the new gray value of pixel in position (*x*, *y*), *OrgGrayValue*(*i*, *j*) is the mask value. Figure 1 is the original image and Figure 2 is the blurred image after applying the above rod-shape Gaussian filter. After this blurring process, a blurred image is generated and served as the input image for the EM algorithm.

In applying of the EM algorithm to the blurred image, the value of $\{C_{ijmn}\}$ array is set to be the same as the Gaussian filer, the incomplete data set $\{X_{ij}\}$ is the blurred image, and the complete data set $\{\lambda_{mn}\}$ is then the original image. We start the EM algorithm by arbitrary assigning the value of complete date set $\{\lambda_{mn}\}$, after the iteration of E-step and M-step discussed in the previous section, a deblurred image is obtained. Figure 3 is the deblurred result after 100 iterations of the EM algorithm.

Figure 1 The original Lena image.

Figure 2 The blurred version of Lena image.

Figure 3 The deblurred result obtain after 100 iterations of the EM algorithm.

Figure 5 The result obtained from simple duplication method.

Scale change

In this part, the EM algorithm is applied to the image scaling application. A scale down version of the original Lena image is generated first. A 3 by 3 square area in the original image is mapped to a single pixel in the reduced version, with the grayscale value as the average of the pixels contained within the square, i.e.

$$NewPixelValue(x,y) = \frac{1}{9}\sum_{i=3x-2}^{3x}\sum_{j=3y-2}^{3y} OrgPixelValue(i,j),$$

where $NewPixelValue(x, y)$ represents the new scale down pixel value in position (x, y); $OrgPixelValue(i, j)$ is the original pixel in position (i, j). The size of new image is 1/9 of the original image. The reduced and blurred version of the Lena image is shown in Figure 4. This image is then served as the input image for the EM, simple duplication, and linear interpolation procedures for enlarging to the original scale. The enlarged and deblurred image using these three different methods are shown in Figure 5, 6 and 7. The high frequency characteristic obtained from the EM method is clearly superior to those obtained from two other methods.

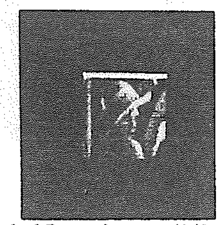

Figure 4 The scaled Lena image (1/9 of original image).

Figure 6 The result obtained from the linear interpolation method.

Figure 7 The result obtained from the EM method.

4. CONCLUSIONS

In this paper, the application of the EM algorithm to image contrast and resolution enhancement is proposed. The EM algorithm is a simple numerical method and can be easily adapted to a large class of image processing applications. The EM algorithm can incorporate the potentially complex nature of various image formation processes into a single probability density array and iteratively converge monotonically to a global maximum. The image quality obtained from using the EM method is superior to that of convention ones and can be applied to a broad class of problems. The efficiency of the EM algorithm has yet to be addressed. The rate of convergence can be painfully slow if a lot of data are unobserved in the EM paradigm. A more efficient iteration method accelerating the convergence process is the topic to be discussed in the future research.

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