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A Modified Morphological Corner Detector

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Abstract*

In this paper we propose a modified morphological corner detection method which can find convex and concave significant points using simple integer computation. We use the morphological operator $A - A \circ B$ *as a useful extractor to preserve some connected regions for detecting convex corners and use the morphological operator* $A \bullet B - A$ *dilated by a small structuring element to maintain some connected regions for detecting concave corners. For complementary, maximizing distance measurement between the pixel belong to a boundary segment in two contiguous significant points and the chord connecting these two contiguous significant points can be applied to remedy the loosing corner due to the shape of the chosen structuring element.*

1. Introduction

Corners are very useful features in image matching and shape analysis [1]. Traditionally, a point is declared as a corner point if, at this point, the object boundary makes discontinuous changes in direction or the curvature of the boundary is above some threshold. Many approaches [2-10] have been developed for corner detection. They are all based on the analysis of chain-coded boundaries of objects. Most of them involve complex floating point computation. Sarkar's [10] proposes a simple algorithm to detect significance vertices, however his method is sensitive to boundary noise or boundary distortions. In summary, the above boundary-based corner detection methods suffer either the dependence on the correctness of region segmentation or the susceptivity of noise.

With computational simplicity and effectiveness, mathematical morphology has been applied successfully to image and signal processing [12][13]. The existent morphological corner detectors [14, 15] are derived from Meyer's peak and valley extractors [16]. It should be noted that the result of peak or valley extractor of an object consists of areas around corner points. Therefore, it is our goal in this paper to modify the existent morphological corner detectors such that they can find the actual corner points.

The paper is organized as follows. In section 2, we will introduce the elementary theorem of mathematical morphology. In section 3, we will describe the method we proposed for corner detection. In section 4, we discuss the effects of different type structuring elements and the size of structuring element. In our experiment, the results show that the method is effective in detecting corners from experimental images. Finally, the conclusions will be given in section 5.

2. Mathematical Morphology

Mathematical morphology is a well-known tool in image processing [12, 13]. The basic operations in the algebraic framework of mathematical morphology are dilations and erosions, each associated with a structuring element. These operations can be defined on Euclidean space with arbitrary dimension. In this paper, we confine ourselves to define them on the discrete Euclidean plane Z^2.

Let A be a subset of Z^2. The set $-A = \{-a | a \in A\}$ is usually called the reflected set of A. If x is a point in Z^2, then the set

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$A_x = \{a + x | a \in A\}$ is called the translation of A by x.

Definition 2.1. Let A and B be two subsets of Z^2 The *dilation* of A by B, written as $A \oplus B$, is given by

$$A \oplus B = \bigcup_{b \in B} A_b .$$

The *erosion* of A by B, written as $A \ominus B$, is given by

$$A \ominus B = \bigcap_{b \in B} A_{-b} .$$

Then, the *closing* of A by B, written as $A \bullet B$, is given by $A \bullet B = (A \oplus B) \ominus B$. The *opening* of A by B, written as $A \circ B$, is given by $A \circ B = (A \ominus B) \oplus B$.

The subset B used in the above definition is called a *structuring element*. In most applications, a structuring element is chosen to have small size and simple shape. Many useful properties of dilations, erosions, closings, and openings can be found in the pioneer works of Matheron [11] and Serra [12]. To our application, we notice that, if B is a disk-shaped structuring element, the effect of $A \circ B$ is to smooth away some convex portions of A. While the effect of $A \bullet B$ is to fill in some concave portions of A. Therefore, as in the works of Noble [14] and Shapiro et al. [15], it is intuitively to use the set difference $A - A \circ B$ (this is called the peak extractor when A is a numerical function [12]) to locate convex corners and use $A \bullet B - A$ (this is called a valley extractor when A is a numerical function [12]) to locate concave corner points. However, as we pointed out before, the results of $A - A \circ B$ and $A \bullet B - A$ consist of areas around corner points not only corner points themselves. Furthermore, since $A \bigcap (A \bullet B - A) = \varnothing$, if we are required that all corner points of A must lie in A, then $A \bullet B - A$ is obviously not the desired corner detector. In a word, the localization ability of the existent morphological corner detection must be improved in order to find the actual positions of corner points.

3. Proposed method

Let A be a binary image, i.e., a subset of Z^2, and B be a disk-shaped structuring element. In this section, we will first modify the morphological operator $A - A \circ B$ to find convex corner points. Then, we will modify the operator $A \bullet B - A$ to find concave corner points.

A. Convex corner detection

Before describing the proposed convex corner detection, we give the following definition first.

Definition 3.1. Let A be a binary image, N a neighborhood of the origin. For each point p in A, the *N-hit number* of p, written as $H_{N \uparrow A}(p)$, is defined by

$$H_{N \uparrow A}(p) = |N_p \cap A|.$$

Keeping this definition in mind, we observe that a boundary point p in a binary image A is a convex corner point then the number $H_{N \uparrow A}(p)$ should be a local minimum around its neighbor for a suitable neighborhood N. But, it is not true that a boundary point with a local minimal hit number is a convex corner point. To overcome this cumbersome, in the first step of our algorithm, we will use the morphological operator $A - A \circ B$ to locate the approximate positions of convex corner points. In the second step of our algorithm, we will search the boundary of each connected component of $A - A \circ B$ points with local minimal hit numbers then choose one of these points as the convex corner point.

Algorithm 1:

Input: Binary image A;
Neighborhood of the origin N;
Disk-shaped structuring element B;

Output: Convex corner points of A;

Step 1. Form the set difference $A - A \circ B$.

Step 2. For each connected component R of $A - A \circ B$ do

find the boundary curve C which is the intersection of the boundary of A and that of R;

chain-coded the curve C by $p_1, p_2, \ldots, p_n$, for some n;

compute the N-hit number $H_{N \uparrow A}(p_i)$ for each i;

search for points with local minimal hit numbers;

Index of p_i	1	2	3	4	5	6	7	8	9	10
$H_{N\uparrow A}(p_i)$	6	7	4	6	6	4	6	6	6	6

Table 1. *N*-hit number string of Fig. 1.

choose the point whose index is as close as to $n/2$ as the convex corner point.

Fig. 1 shows an acute part detected by $A - A \circ B$ with a large disk-shaped structuring element with size 9×9 and Table 1 shows the corresponding *N*-hit numbers of points on curve C using a neighborhood of 3×3 square centered at origin. Positions with local minimal of *N*-hit numbers are in the indices of 3 and 6 that are labeled with circle, determined by algorithm Step 2. In our experiment, the index 6 is as close as to 5 ($n/2$). Thus, we choose p_6 as the convex corner point.

This corner detector uses simple integer operations instead of complex floating point computations to find convex significant corner points, it is simple and fast.

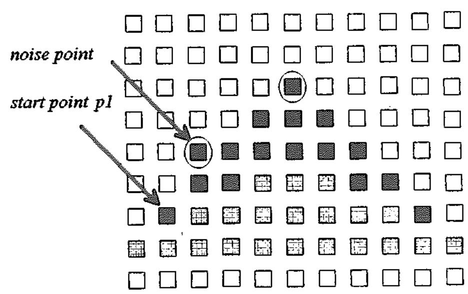

Fig. 1. A connected region of $A - A \circ B$.

B. Concave corner detection

Similar to the observation in subsection A, a boundary point p in a binary image A is a concave corner point then the number $H_{N\uparrow A}(p)$ should be a local maximum around its neighbor for a suitable neighborhood N. But, the converse is not true. Moreover, as we pointed out in the section 2, $A \cap (A \bullet B - A) = \varnothing$. That is, the boundary of A does not belong to the set $A \bullet B - A$. This situation is illustrated in Fig. 2. Thus, we modify the operator $A \bullet B - A$ to be $(A \bullet B - A) \oplus E$, where E is the rhombus structuring element. Then the true concave corner can be enforced to locate on the chain code of the boundary curve C that is the intersection of the boundary of A and that of a connected region R obtained from $(A \bullet B - A) \oplus E$. In the second step of our algorithm, we will search the boundary of each connected component of $A \bullet B - A$ to find points with local maximal hit numbers then choose one of these points as the concave corner point.

Algorithm 2:

Input: Binary image A;
Neighborhood of the origin N;
Disk-shaped structuring element B;
Rhombus structuring element E;

Output: Concave corner points of A;

Step 1. Form the image $(A \bullet B - A) \oplus E$.

Step 2. For each connected component R of $(A \bullet B - A) \oplus E$ do
find the boundary curve C which is the intersection of the boundary of A and that of R;
chain-coded the curve C by $p_1, p_2, \ldots, p_n$, for some n;
compute the *N*-hit number $H_{N\uparrow A}(p_i)$ for each i;
search for points with local maximal hit numbers;
choose the point whose index is as close as to $n/2$ as the concave corner point.

Table 2 shows the corresponding *N*-hit number string of the chain code of boundary curve in Fig. 2. Position with the maximal hit number and as close as to $n/2$ is in the index of 3 which is labeled with circle. By our algorithm, the detected concave corner point is p_3.

Moreover, the detection phase of concave corners can be parallel computed with the detection phase of convex corners. The complete architecture of our corner detection method is shown in Fig. 3. In our algorithm, the procession only involved the

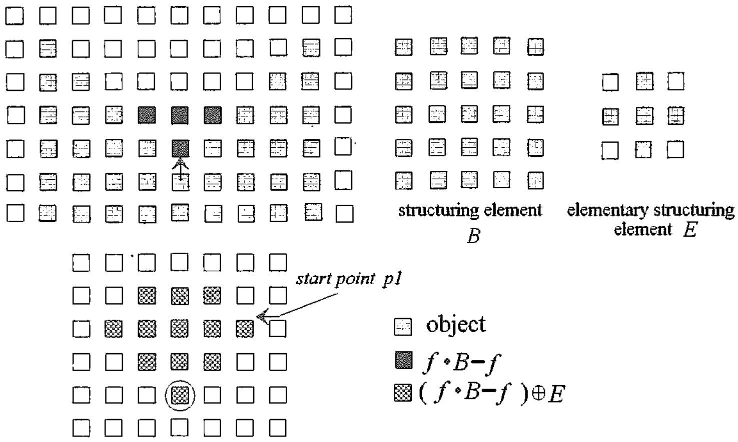

Fig. 2. An example of concave corner detection phase.

Index of p_i	1	2	3	4	5
$H_{N\uparrow A}(p_i)$	6	6	8	6	6

Table 2. The concave number string of a local concave region in Fig. 3.

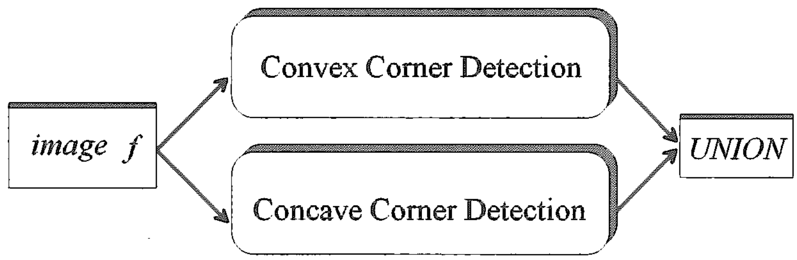

Fig. 3. The architecture of our proposed method.

morphological operators and the simple integer computation. As we know, morphological operator can be implanted to parallel machine. Then the computation time will decrease respectively.

4. Experimental Results and Discussion

In the application of digital image processing, it is a difficult skill to choose a suitable structuring element to fit working purpose. In our observation, we choose different structuring elements for testing such as square and rhomboid. Square can preserve rhombus convex angles and rhombus concave angles, but it will lose right angles. On the other hand, rhomboid can maintain right convex angles and right concave angles, but lose rhombus angles. In our experiment, Fig. 4(a) is the original object image, Fig. 4(b) and 4(c) show the result of corner detection using the square and the rhombus structuring elements with the size of 7 × 7, and the Fig. 4(d) show the union of Fig. 4(b) and Fig. 4(c).

Besides, our algorithm is applied to two digital binary images chromosome and eight images. The corresponding corner images are shown in Fig. 5(a) and 5(b) resulted from the union of two corner images in which one using square and another using rhombus structuring element. Obviously, our experimental results show that our method can detect significant corner points of the stereoscope image, such as Fig. 5(a) and Fig. 5(b). We also observe that as the size of structuring element increasing, the corners detect from our method will more precisely, but will increase computation. It is because that the corner parts detect from morphological operators representing more information about corners. However, as the size of structuring element is too large, the geometric property of local convex or concave region was destroyed by the size of structuring element. Some redundant corners may appear. In this situation, a candidate corner point should be removed if it and its two neighborhoods on the global chain code of object are in a line.

Unfortunately, morphological operators $A - A \circ B$ and $A \bullet B - A$ loose connected component when the corner point has the angle of 135°. For the purpose of resolving the weakpoint of square or rhomboid structuring element, we choose circular structuring element to preserve local convex or concave parts. We applied our method to Fig. 6(a) a fighter image and Fig. 7(a) a tank image by a circular structuring element with size 7 × 7. Fig. 6(b) and Fig. 7(b) show the effective results of corner images.

Moreover, we join some secondary representative points between two contiguous significant corners, as considered in the shape representation. We use the maximum Euclidean distance as the measurement of secondary representative points. The intuitive concept is, for each pixel belong to a boundary segment, we compute the perpendicular Euclidean distance between the pixel and the chord connecting two contiguous significant corner points of the boundary segment. Let c_1 and c_2 be two contiguous significant corners detected from the above method, and $p_1, p_2, \cdots, p_{l-1}, p_l$ be the boundary points from c_1 to c_2, where $p_1 = c_1$ and $p_l = c_2$, then the Euclidean distance from p_i to the chord connecting c_1 and c_2 is denoted as $d(p_i, \overline{c_1c_2})$. If $d(p_{i-1}, \overline{c_1c_2}) > d(p_i, \overline{c_1c_2}) < d(p_{i+1}, \overline{c_1c_2})$ and $d(p_i, \overline{c_1c_2}) > t$, where t is a threshold reflecting the degree of the importance of the secondary representative corner. In other words, for polygonal approximation in shape analysis, the threshold t can be selected with smaller number to minimize the integral square $E_2 = \sum_i e_i^2$, where e_i denotes the error between a boundary segment and a line approximation. It is reasonable to select a lower threshold as the object is small and to select a higher threshold as the object is big. In our experiment, we measure the merit E_2 of some algorithms on chromosome image in Table 3. The result shows that our method obtain a smaller error than others. Finally, some experimental results are shown in Fig. 8 (a), Fig. 8 (b), and Fig. 8 (c).

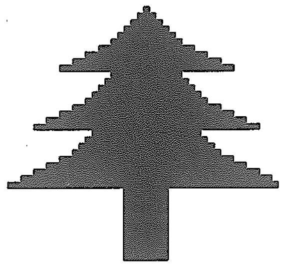

Fig. 4(a) A binary original tree image.

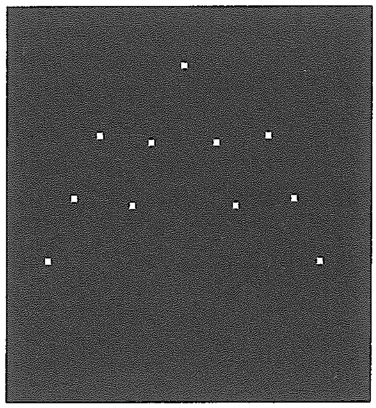

Fig. 4(b) The corner image detected by square structuring element.

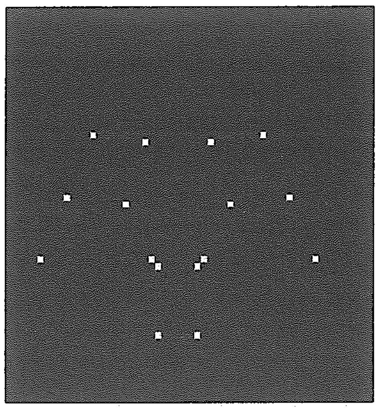

Fig. 4(c) The corner image detected by rhombus structuring element.

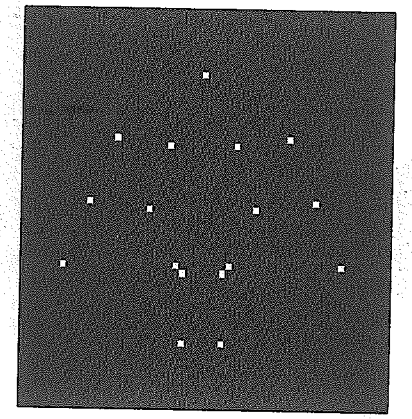

Fig. 4(d) The union of Fig. 4(b) and Fig. 4(c).

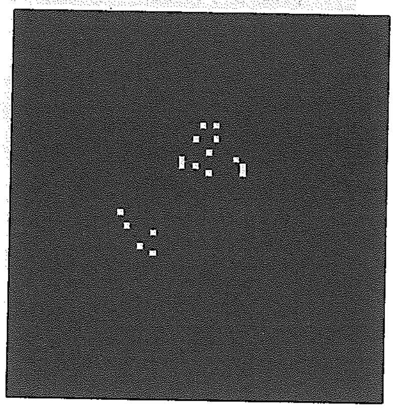

Fig. 5(a) The corner image of chromosome image.

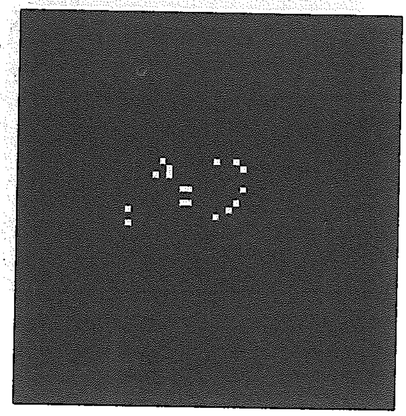

Fig. 5(b) The corner image of eight image.

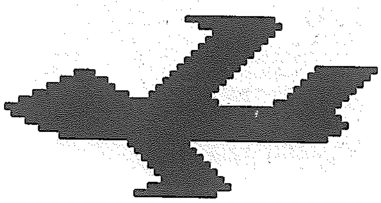

Fig. 6(a) The fighter image.

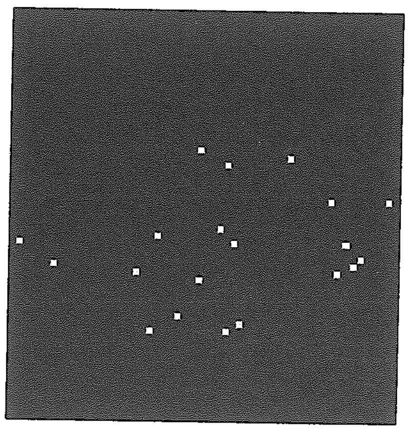

Fig. 6(b) The corner image of Fig. 6(a).

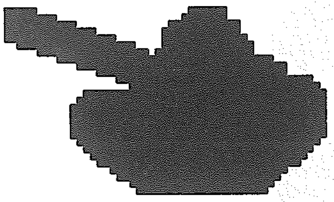

Fig. 7(a) The tank image.

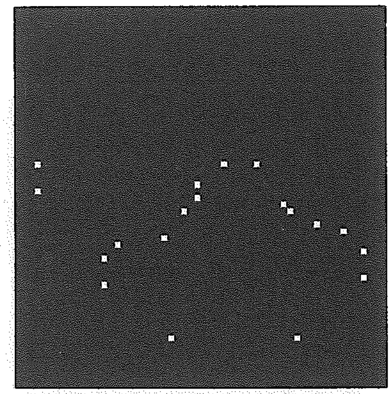

Fig. 7(b)The corner image of Fig. 7(a)

Algorithms	Total Error $E_2 = \sum_i e_i^2$
Rosenfled-Johnston [2] m=6	14.131202
Rosenfled-Weszka [3] m=6	15.473706
Freeman-Davis [4] s=3, m=2	17.329360
Sankar-Sharma [5]	24.873161
Teh-Chin (k-cosine) [6]	10.081549
Teh-Chin (k-curvature) [6]	7.481549
Teh-Chin (1-curvature) [6]	7.481549
M. M. corner detector t=1	7.286041

Table. 3.

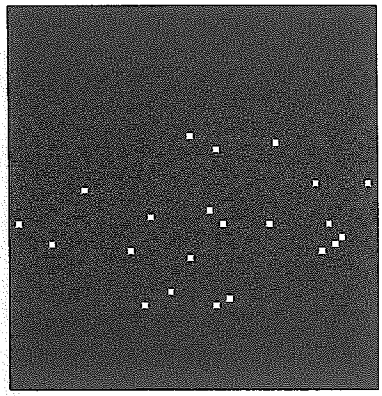

Fig. 8(a)

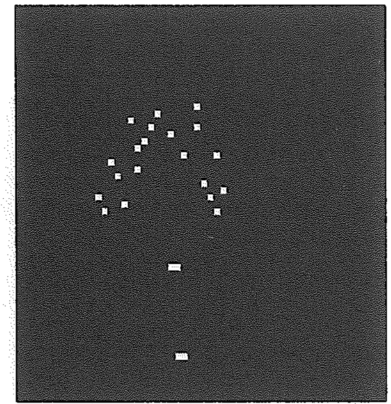

Fig. 8(b)

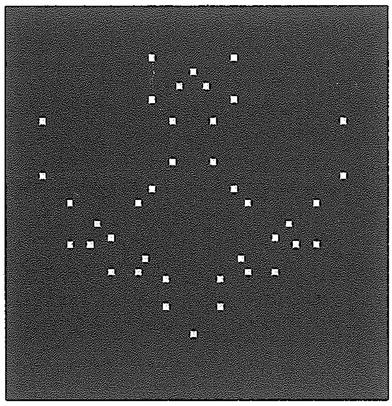

Fig. 8(c)

Fig. 8 (a). The corner image of Fig. 6(a); Fig.8 (b).The corner image of leaf image with threshold t=2.0; Fig. 8 (c). The corner image of civil aircraft image for shape representation with threshold t=1.0.

5. Conclusions

In this paper, we have presented a corner detection method using mathematical morphological operators and simple integer operation. As we know, corners are usually divided into convex and concave corners. At first, we detect convex corners and concave corners concurrently using morphological operators as useful tools to extract connected regions about geometrical corners. Then for the purpose of indicating good location, a corner point on object boundary is thought as the position located on the divided line of equal angle and the position with extreme value of N-hit number. In our experiment, we exhibit satisfied experiment results in detecting significant points and providing good approximation on shape description.

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