Modeling CDPD Channel Holding Times*

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Abstract— **Cellular Digital Packet Data (CDPD) provides wireless data communication services to mobile users by sharing unused RF channels with AMPS on a non-interfering basis. To prevent interference on the voice activities, CDPD makes forced hop to a channel stream when a voice request is about to use the RF channel occupied by that channel stream. The number of forced hops is affected by the voice channel selection policy. We propose an analytic model to investigate the CDPD channel holding time for the the least-idle and random voice channel selection policies. Under various system parameters and voice channel selection policies, we provide guidelines to reduce the number of forced hops.**

Key Words: Cellular Digital Packet Data, Voice Channel Selection Policy, Forced Hop, Channel Holding Times, AMPS, .

EDICS Number: CL1.2.2 Wireless Networks, CL1.2.5 Wireless Personal Communications

I. INTRODUCTION

Cellular Digital Packet Data (CDPD) [2] provides wireless data communication services to mobile users by sharing the radio equipment and unused RF channels with Advanced Mobile Phone Service (AMPS).

A CDPD subscriber uses the mobile end system to communicate with the CDPD mobile data base station (MDBS) through a 19.2 kb/s duplex wireless link (referred to as a *CDPD channel stream*). The MDBS is responsible for detailed control of the radio interface, which shares a subset of RF frequencies of the associated AMPS base station (BS). To determine the frequencies in the CDPD frequency pool and prevent both systems from choosing the same frequency at the same time, the MDBS employs a sniffer that periodically scans the shared channels to identify the availability status of these RF channels. MDBS transmits packet data over idle AMPS channels, and autonomously switches to another channel when the current channel is about to be assigned for voice usage. With the sniffer mechanism, AMPS does not notice the existence of CDPD who shares the AMPS resources.

When a voice call arrives at an AMPS BS, the BS selects an idle RF channel to serve this incoming voice request. If this RF channel is occupied by a CDPD channel stream, the MDBS must relinquish this RF channel within 40 ms. This action is called *forced hop*. The MDBS then tries to re-establish the forced-hopped channel stream on another idle RF channel. If no such channel is available, the forced-hopped channel stream enters a *blackout* period [1] until an idle RF channel is re-assigned to this channel stream.

Since switching a CDPD channel stream from an RF frequency to another is an expensive operation, it is important to exercise an appropriate voice channel selection strategy (under the constraint that AMPS does not notice the existence of CDPD) to minimize the CDPD channel switching; that is, to maximize the CDPD channel holding time t_a which is the period that an RF channel is utilized by a CDPD channel stream before a forced hop occurs on this CDPD channel stream.

We investigate two voice channel selection policies. In the least-idle policy, when a voice request arrives, the least idle channel (in terms of voice usage) is selected. In the random policy, an arbitrary idle channel is selected for the voice request. We show how t_a is affected by the voice channel selection policies under various system parameters.

II. THE ANALYTIC MODEL

We propose an analytic model to study the CDPD channel holding time t_a for least-idle and random voice channel selection strategies. Following the studies in [1] and [4], planned hop (a CDPD mechanism that periodically switches a CDPD channel stream from one idle RF channel to another) is not considered. The reader is referred to [3] for more details about the impact of planned hop.

Suppose that the voice call arrivals form a Poisson process. Let x_i be the interarrival time between the $i-1$st and the ith voice calls (see Figure 1). Then x_i is exponentially distributed with rate λ. Suppose that a voice channel is released in the time interval x_i. Let R be the period between the release of an RF channel and when the next voice call arrives. In Figure 1, the CDPD channel holding time t_a is expressed as

$$t_a = R + x_{i+1} + \ldots + x_k$$

Since the channel release is a random observer of the time interval x_i, from [5], R has the same distribution as x_i. Let α be the probability that a CDPD channel stream is forced hopped by a voice arrival. The density function $f_a(t_a)$ of t_a is

$$\begin{aligned} f_a(t_a) &= \sum_{k=1}^{\infty} \int_{x_1=0}^{t_a} \int_{x_2=0}^{t_a-x_1} \cdots \int_{x_k=0}^{t_a-x_1-x_2-\cdots-x_k} \\ &\quad (1-\alpha)^{k-1}\alpha \\ &\quad \times \prod_{i=1}^{k} (\lambda e^{-\lambda x_i}) dx_k dx_{k-1} \ldots dx_1 \end{aligned}$$

and its Laplace transform is

$$\begin{aligned} f_a^*(s) &= \int_{t_a=0}^{\infty} f_a(t_a) e^{-st_a} dt_a \\ &= \sum_{k=1}^{\infty} (1-\alpha)^{k-1}\alpha \left(\frac{\lambda}{s+\lambda}\right)^k \\ &= \frac{\alpha\lambda}{s+\alpha\lambda} \end{aligned}$$

The mean CDPD channel holding time $E[t_a]$ is expressed as

$$E[t_a] = (-1)\frac{df_a^*(s)}{ds}\bigg|_{s=0} = \frac{1}{\alpha\lambda} \quad (1)$$

The probability π_i that i channels are used by voice users can be obtained from an M/G/C/C queuing model. Suppose that the voice channel holding times (the period that an RF channel is occupied by a voice call) have a general distribution with mean $1/\mu$. Then the voice offered load to an AMPS BS is $\rho = \lambda/\mu$. For $0 \le i \le N$,

$$\pi_i = \frac{\frac{\rho^i}{i!}}{\sum_{j=0}^{N} \frac{\rho^j}{j!}}$$

where N is the number of RF channels in the BS.

Under the random voice channel selection strategy, when i channels are occupied by voice users, the probability that a CDPD channel stream is forced hopped is $\frac{1}{N-i}$, and

$$\alpha = \sum_{i=0}^{N-1} \left(\frac{\pi_i}{N-i}\right) \quad (2)$$

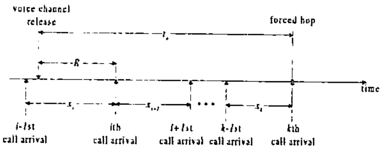

Fig. 1. The Timing Diagram

To derive $E[t_a]$ for the least-idle voice channel selection strategy, we first show that when a voice call request arrives, no CDPD channel stream is forced hopped if the number of RF channels not used by voice is larger than the number N_{CDPD} of the maximum CDPD channel streams.

Theorem 1: Suppose that the voice channel assignment is least-idle, and a blackout CDPD channel stream always occupies an RF channel immediately after the RF channel is released from voice usage. Then

(I) The CDPD channel streams always occupy the most idle RF channels immediately after a voice call request arrives.

(II) When a voice call request arrives, no CDPD channel stream is forced hopped if the number of RF channels not used by voice is larger than N_{CDPD}.

Proof: During the steady-state operation of the system, all RF channels are busy for voice usage from time to time. Suppose that all RF channels are busy at time t_0, and after t_0, the ith voice call request arrives at time t_i. We prove by induction on t_i that immediately after t_i, hypotheses (I) and (II) hold.

Basis: Consider the period $T = (t_0, t_1]$. Suppose that there are j voice call completions during T (where $0 \le j \le N$). Let $l = \min[j, N_{CDPD}]$. Since a blackout CDPD channel stream always occupies an RF channel immediately after the RF channel is released from voice usage, the first l released voice RF channels are occupied by the CDPD channel streams during T, and they are the l most idle RF channels. Thus, hypothesis (I) holds immediately before t_1. When the first voice request arrives at time t_1, an RF channel not used by voice is selected based on the least-idle policy. If $j = 0$, then the voice request is rejected. If $0 < j \le N_{CDPD}$, then after the voice channel assignment, hypothesis (I) still holds, and $N_{CDPD} - j + 1$ CDPD channel streams are blacked out. If $j > N_{CDPD}$, then from the least-idle policy, none of the N_{CDPD} most idle RF channels (in terms of the voice usage) is selected to serve the voice request, and both hypotheses (I) and (II) hold.

Inductive Step: Suppose that hypotheses (I) and (II) hold immediately after t_i. We show that these hypotheses also hold immediately after t_{i+1}.

Consider the period $T^* = (t_i, t_{i+1}]$. Suppose that there are j_i RF channels not used by voice immediately after t_i. During T^*, j^* RF channels are released from voice usage. Three cases are considered:

- Case I ($j_i > N_{CDPD}$): Since hypothesis (I) holds immediately after t_i, the N_{CDPD} most idle RF channels are occupied by all CDPD channel streams. At t_{i+1}, one of the remaining $j_i + j^* - N_{CDPD}$ RF channels is selected to serve the i + 1st voice request based on the least-idle policy, and both hypotheses (I) and (II) hold immediately after t_{i+1}.
- Case II ($0 \le j_i \le N_{CDPD}$ and $j_i + j^* > N_{CDPD}$): Since a blackout CDPD channel stream always occupies an RF channel immediately after the RF channel is released from voice usage, all j_i RF channels are occupied by the CDPD channel streams at time t_i, and immediately before t_{i+1}, the N_{CDPD} most idle RF channels are occupied by all the CDPD channel streams. Since $j_i + j^* > N_{CDPD}$, the situation is exactly the same as that in Case I at time t_{i+1}. Following the same reasoning, both hypotheses (I) and (II) hold immediately after t_{i+1}.
- Case III ($0 \le j_i + j^* \le N_{CDPD}$): It is apparent that hypothesis (I) holds immediately after t_{i+1}. If $j_i + j^* = 0$, then the i + 1st voice request is rejected. Otherwise, the voice request is accommodated and $N_{CDPD} - j_i - j^* + 1$ CDPD channel streams are blacked out. **QED**

The study in [4] observed that if the least-idle voice channel selection policy is used, then the CDPD channel selection policy (least-idle, most-idle, clockwise, and so on) does not affect $E[t_a]$. Theorem 1 (I) formally proves that in the steady state, the least-idle voice channel selection policy always results in one CDPD channel selection policy (i.e., the most-idle policy).

If the voice channel selection strategy is least-idle, and i channels are occupied by voice users when a voice request arrives (where $N - N_{CDPD} \le i \le N - 1$), then from Theorem 1, the probability that a CDPD channel stream is blacked out is $\frac{1}{N-i}$ and

$$\alpha = \sum_{i=N-N_{CDPD}}^{N-1} \left(\frac{\pi_i}{N-i} \right) \qquad (3)$$

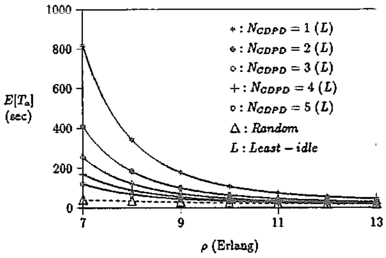

Fig. 2. The effect of N_{CDPD} ($N = 15$, $\lambda = 0.172$ *calls/sec*)

III. NUMERICAL RESULTS

The results based on our analytic model are consistent with the results in [4]. Given $N = 15$, $N_{CDPD} = 1$, $\lambda = 0.172$ calls/sec and $\rho = 9.01$ Erlangs, we have $E[t_a] = 174.567$ seconds for the least-idle policy, while $E[t_a] = 164.55$ seconds in [4] (the discrepancy is within 6%). On the other hand, we generalize the results of [4] by allowing $N_{CDPD} > 1$. Our analysis indicates the following.

The Effect of N_{CDPD}: Figure 2 plots $E[t_a]$ as a function of the voice traffic ρ with different N_{CDPD} values, where $N = 15$ and $\lambda = 0.172$ calls/sec. It is clear that $E[t_a]$ is a decreasing function of ρ. In addition, when the voice channel selection strategy is least-idle, $E[t_a]$ decreases as N_{CDPD} increases. When N_{CDPD} increases, α increases (see (3)), and $E[t_a]$ decreases (see (1)). This effect becomes significant when N_{CDPD} or ρ is small. For example, when $\rho = 7$, if N_{CDPD} decreases from 2 to 1, $E[t_a]$ is increased by 100.73%. If N_{CDPD} decreases from 3 to 2, $E[t_a]$ is increased by 61.91%. However, the CDPD throughput increases as N_{CDPD} increases. Thus, the selection of the N_{CDPD} value becomes a trade-off. Our result is different from the conclusion in [4] where the authors claimed that $E[t_a]$ for multiple CDPD channel streams should be the same as that in the single channel stream scenario. Figure 2 also shows that $E[t_a]$ for the least-idle strategy is larger than the random strategy. This phenomenon is also shown in Figures 3 and 4. In fact, the $E[t_a]$ values are the same for both least-idle and random policies if $N_{CDPD} = N$ (see (2) and (3)).

The Effect of N: With $N_{CDPD} = 3$ and $\lambda = 0.172$ calls/sec, Figure 3 indicates that by increasing N, $E[t_a]$ increases. A larger N provides more RF channels to the system, which reduces the opportunity that a CDPD channel stream is forced hopped, and $E[t_a]$ increases. This effect is significant for a large

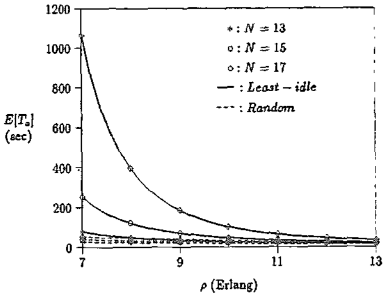

Fig. 3. The effect of N ($N_{CDPD} = 3$, $\lambda = 0.172$ *calls/sec*)

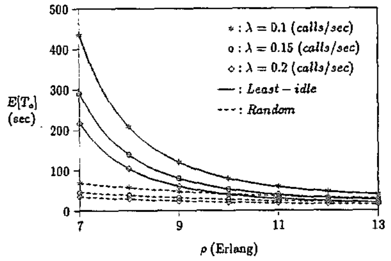

Fig. 4. The effect of λ ($N_{CDPD} = 3$, $N = 15$)

N and a small ρ.

The Effect of λ: Figure 4 shows that for a fixed ρ value, $E[t_a]$ is a decreasing function of the voice call arrival rate λ. This phenomenon is a direct consequence of (1), which indicates that $E[t_a]$ is inversely proportional to λ. When the voice call arrival rate λ increases, an active CDPD channel stream is more likely to be preempted, which results in decreasing $E[t_a]$. The effect is significant for small λ.

IV. CONCLUSIONS

This paper studied the effects of least-idle and random voice channel selection policies on the mean CDPD channel holding time $E[t_a]$. We proved that the least-idle voice channel selection policy always results in one CDPD channel selection policy, and when a voice call request arrives, no CDPD channel stream is forced hopped if the number of RF channels not used by voice is larger than the number N_{CDPD} of the maximum CDPD channel streams. We have the following specific conclusions:

- To reduce the number of forced hops, the least-idle voice channel selection strategy is superior to the random policy.
- $E[t_a]$ increases as N (the number of the RF channels in a cell) increases and as voice offered load ρ decreases.
- $E[t_a]$ is a decreasing function of N_{CDPD} when the voice channel selection strategy is least-idle. However, $E[t_a]$ is independent of N_{CDPD} when the voice channel selection strategy is random.

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