A Performance Study of CDPD

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Abstract

In this paper, we study the performance of the CDPD system, and show how the CDPD performance is affected by the voice traffic of Cellular-CDPD network.

1 Introduction

Cellular Digital Packet Data (CDPD) offers mobile users access to a low-cost, ubiquitous, wireless data network [11]. There are two basic classes of network entities in the CDPD Network [1]: End Systems (ESs) and Intermediate Systems (ISs). For our purpose, we only consider the mobile parts of the network; that is, we only consider the Mobile End Systems (M-ESs) and the Mobile Data Intermediate Systems (MD-IS). Figure 1 illustrates the CDPD network layer reference model. In this figure, the Mobile Data Base Station (MDBS) is not a network layer entity, but a Layer 2 relay system for user data carriage. The MDBS is a unique and distinct component of the CDPD network, which plays the major role of radio channel assignment to be studied in this paper. A CDPD user communicates with the CDPD network by using the M-ES. The MDBS is responsible for detailed control of the radio interface, such as radio channel allocation, interoperation with cellular voice channel usage and radio media access control. An MD-IS receives data from one correspondent network entity and forwards it to another correspondent network entity. The MD-IS supports user mobility by operating a CDPD-specific Mobile Network Location Protocol (MNLP) to exchange location information of CDPD users.

An M-ES communicates with the corresponding

MDBS by a 19.2 kb/s raw duplex wireless link (which is referred to as a *CDPD channel stream*). A CDPD channel stream can be accessed by several M-ESs in the time sharing fashion.

CDPD channel streams use idle Cellular radios to transmit data to and from M-ESs. However, CDPD systems are designed to use idle Cellular capacity without direct communication with Cellular, and a strict requirement of CDPD is that CDPD transmissions must not interfere with Cellular voice calls. This requirement is achieved as follows. The MDBS continuously monitors idle cellular channels and sends data over an idle voice channel until it is needed for voice traffic. When the channel is being used for voice traffic, the CDPD data stream will "hop" off that channel and onto an idle channel. The M-ESs listen to commands broadcast by the MDBS and follow in concert with CDPD's hopping data channel stream. More details of CDPD can be found in [1, 12].

This paper studies the performance of a CDPD system, and shows how the CDPD performance is affected by the voice traffic of Cellular-CDPD network.

2 The Availability Model

We first study the availability of a CDPD network. That is, we derive the blocking probability p_b that no CDPD channel stream is available when an M-ES attempts to access the network. Budka [2], Jain and Basu [3] and Nanda et al [10] investigated the availability of CDPD channel streams. Although these studies did not consider the Cellular/CDPD user mobility, they did provide a clean and comprehensive availability analysis. Our study is different from these previous studies in two aspects. Firstly we consider

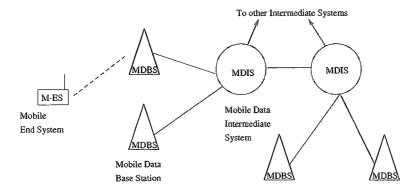


Figure 1: The CDPD Network Layer Reference Model

the Cellular voice user mobility and the CDPD user mobility. Secondly, we consider the CDPD availability to an M-ES, not a CDPD channel stream. Note that the availability of CDPD streams do not have any impact on the CDPD performance during the periods when no M-ESs make requests.

The following notation is used.

- c: the number of radio channels in a cell
- λ, λ_o, λ_h: λ_o is the original voice call arrival rate to a cell, λ_h is the handoff voice call arrival rate, and λ = λ_o + λ_h.
- μ: the voice call completion rate
- η: the cellular (voice) user mobility (or the rate that the user moves out of a cell)
- $\overline{\lambda}$: the request arrival rate of a CDPD user
- μ^* : the CDPD data completion rate
- k: the maximum number of CDPD channel streams allowed in a cell
- N: the expected number of CDPD users in a cell.
- $\overline{\lambda^*} = N\overline{\lambda}$: the expected net CDPD request arrival rate

CDPD user mobility model. We follow the population model proposed in [5]. Let N be the expected number of M-ESs in a cell. Suppose that the residence time of an M-ES in a cell has a general distribution F(t) with mean $\frac{1}{\eta^*}$. In the steady state, the rate of the M-ESs move in a cell equals to the rate of the M-ESs move out of the

cell. In other words, the rate that M-ESs move in a cell is $N\eta^*$. The arrivals of M-ESs can be viewed as being generated from N input streams which have the same general distribution with arrival rate η^* . If N is reasonably large ($N \geq 20$) in a cell, the net input stream is approximated as a Poisson process with arrival rate $N\eta^*$. Thus, the distribution for the M-ES population can be modeled by an $M/G/\infty$ queue with arrival rate $N\eta^*$ and the residence time $1/\eta^*$. Let π_n be the steady state probability that there are n M-ESs in the cell. From the standard technique [4],

$$\pi_n = \frac{N^n e^{-N}}{n!} \tag{1}$$

An important implication of (1) is that the population distribution is independent of the CDPD residence time distributions. The M-ES distribution (i.e., Equation (1)) is used to study the blocking probability p_b for an M-ES. The idea is the following. An MDBS sees n M-ESs with the probability π_n . These M-ESs generate requests with the rate $\lambda^*(n) = n\overline{\lambda}$. Let $p_b^{(n)}$ be the blocking probability of an M-ES when there are n M-ESs in the cell, then in the steady state, the CDPD blocking probability for the cell is

$$p_b = \sum_{k < n < \infty} \pi_n p_b^{(n)} \tag{2}$$

This model captures the user mobility assuming that handoff does not occur frequently. Under the range of the input parameters studied in this paper, this assumption is justified. Since the CDPD users typically use the network for short

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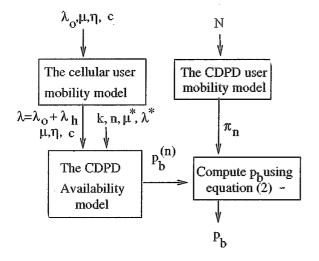


Figure 2: The modeling strategy

messages, it is likely that the service is completed before the user moves out of a cell. If this assumption does not hold, then the complex mobility model (to be described next) should be used.

Cellular user mobility model. We follow the handoff model proposed in [7]. In this model, the handoff rate to a cell is derived as (see Equation (9) in [7])

$$\lambda_h = \frac{\eta(1 - p_{b,v})}{\mu + \eta p_{b,v}} \lambda_o$$

where $p_{b,v}$ is the blocking probability of a voice call, which is computed iteratively by using $\lambda_o, \mu, \lambda_h, \eta$, and c. The details of the iterative algorithm will not be presented in this paper but can be found in [8, 6, 7].

Figure 2 illustrates our modeling strategy. We first use the cellular user mobility model (with parameters λ_o, μ, η , and c) to derive λ_h . Then for $1 \leq n \leq N'$ (where N' >> N), we compute $p_b^{(n)}$ using a CDPD availability model (to be described) with parameters $\lambda, \mu, \eta, c, k, \lambda^*(n), \mu^*$.

To strengthen and simplify our results, we assume that the total number of M-ESs allowed in the system is k. The extension to having more than k M-ESs in the system is trivial but tedious. We will briefly describe the extension at the end of this section. The behavior of the CDPD network is described as follows. Suppose that there are c=20 radio channels in

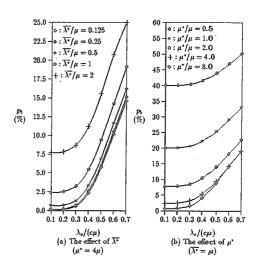


Figure 3: The state diagram (In this figure, λ^* is an abbreviation of $\lambda^*(n)$)

a cell, and the maximum number of the CDPD channel streams allowed in a cell is k = 4. If the number j of the radio channels occupied by the Cellular user conversations is less than c - k = 16, then all 4 CDPD channel streams are available, and up to 4 M-ESs can be in service. The fifth M-ES request will be rejected. Suppose that j = 16 channels are occupied by the cellular users, and 4 CDPD channel streams are occupied by 4 M-ESs. If one Cellular request arrives, then one of the 4 CDPD channel streams will be blacked out, and the radio channel is used by the new voice request. The M-ES using the blacked-out CDPD channel stream will be temporarily out of the service until an occupied radio channel becomes available. In this case, the M-ES served by the stream is forced hopped to one of the other three channel streams. Since CDPD provides packet switching service, it is resonable to assume that the four M-ESs evenly share the three CDPD channel streams. Thus, the behavior of the system can be approximated by a "processor sharing" model. The availability model is described in nine cases illustrated in the state diagram in Figure 3. In this figure, the net CDPD request rate $\lambda^*(n)$ is abbreviated by λ^* . A state (i, j)

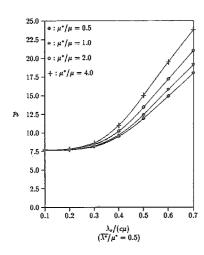


Figure 4: The state diagram (Cont.)

 $(0 \le i \le k \text{ and } 0 \le j \le c)$ represents that there are i CDPD requests granted for service (some of them may be in the waiting state), and j Cellular users in conversation (each of them occupies a radio channel). The state (i, j) where n < i is legal in this model, which implies that some CDPD users issue more than one requests. (At the end of this section, we will show how to model the case where an M-ES can only issue at most one request at a time.) If j > c - k then only c - j CDPD channel streams are available, and k-c+j CDPD channel streams are blacked out. In this case, if i > c - j then c - j M-ESs are in service and i - c + j M-ESs are in the waiting state. In all cases, the process moves from (i, j) to (i, j + 1) with rate λ , and moves from (i, j) to (i, j-1) with rate $j\mu$. (Since there are j voice calls in service at state (i, j), the first voice call will be completed with rate $j\mu$.) In all cases, the process moves from (i, j) to (i + 1, j)with rate λ^* . For j < c-i (where $1 \le i \le k$), i CDPD requests are served by radio channels, and the first request will be completed with the rate $i\mu^*$. Thus, the process moves from state (i, j) to (i - 1, j) with rate $i\mu^*$ (see Figure 3 Cases (A)-(F)). For $j \geq c-i$, (where $1 \leq i \leq k$), only (c-j) radio channels are available to serve the CDPD users, and the process moves from (i,j) to (i-1,j) with rate $(c-j)\mu^*$ (see Figures 3 and 4 Cases (G)-(K)). Let $p_{i,j}$ be the probability that the process is in state (i, j). From these state diagrams, we write the system of balance equations for the probability $p_{i,j}$.

Case A. i = 0, j = 0 (see Figure 3 (A))

$$(\lambda^* + \lambda)p_{0,0} = \mu^* p_{1,0} + \mu p_{0,1}$$

$$\Rightarrow p_{0,1} = \frac{\lambda^* + \lambda}{\mu} p_{0,0}$$

$$- \frac{\mu^*}{\mu} p_{1,0}$$
 (3)

Case B. 0 < i < k, j = 0 (see Figure 3 (B))

$$(\lambda^* + \lambda + i\mu^*)p_{i,0} = (i+1)\mu^*p_{i+1,0} + \mu p_{i,1}$$

$$+ \lambda^*p_{i-1,0}$$

$$\Rightarrow p_{i,1} = \frac{\lambda^* + \lambda + i\mu^*}{\mu} p_{i,0} - \frac{(i+1)\mu^*}{\mu} p_{i+1,0} - \frac{\lambda^*}{\mu} p_{i-1,0}$$
(4)

Case C. i = k, j = 0 (see Figure 3 (C))

$$(\lambda + k\mu^*)p_{k,0} = \mu p_{k,1} + \lambda^* p_{k-1,0}$$

$$\Rightarrow p_{k,1} = \frac{\lambda + k\mu^*}{\mu} p_{k,0} - \frac{\lambda^*}{\mu} p_{k-1,0}$$
 (5)

Case D. i = 0, 0 < j < c (see Figure 3 (D))

$$(j\mu + \lambda^* + \lambda)p_{0,j}$$

$$= \lambda p_{0,j-1} + \mu^* p_{1,j} + (j+1)\mu p_{0,j+1}$$

$$\Rightarrow p_{0,j+1} = \frac{j\mu + \lambda^* + \lambda}{(j+1)\mu} p_{0,j} - \frac{\lambda}{(j+1)\mu} p_{0,j-1}$$

$$- \frac{\mu^*}{(j+1)\mu} p_{1,j}$$

or for $i = 0, 1 < j \le c$,

$$p_{0,j} = \frac{(j-1)\mu + \lambda^* + \lambda}{j\mu} p_{0,j-1} - \frac{\lambda}{j\mu} p_{0,j-2} - \frac{\mu^*}{j\mu} p_{1,j-1}$$
(6)

Case E. 0 < i < k, 0 < j < c - i (see Figure 3 (E))

$$(j\mu + \lambda^* + \lambda + i\mu^*)p_{i,j}$$

$$= \lambda p_{i,j-1} + (i+1)\mu^* p_{i+1,j}$$

$$+ (j+1)\mu p_{i,j+1} + \lambda^* p_{i-1,j}$$

After re-arrangement,

$$p_{i,j+1} = \frac{j\mu + \lambda^* + \lambda + i\mu^*}{(j+1)\mu} p_{i,j}$$

$$- \frac{\lambda}{(j+1)\mu} p_{i,j-1} - \frac{(i+1)\mu^*}{(j+1)\mu} p_{i+1,j-1}$$

$$- \frac{\lambda^*}{(j+1)\mu} p_{i,j}$$

or for $0 < i < k, 1 < j \le c - i$

$$p_{i,j} = \frac{(j-1)\mu + \lambda^* + \lambda + i\mu^*}{j\mu} p_{i,j-1}$$

$$- \frac{\lambda}{j\mu} p_{i,j-2} - \frac{(i+1)\mu^*}{j\mu} p_{i+1,j-1}$$

$$- \frac{\lambda^*}{j\mu} p_{i-1,j-1}$$
 (7)

Case F. $i = k, 0 < j \le c - k$ (see Figure 3 (F))

$$(j\mu + \lambda + k\mu^*)p_{k,j} = \lambda p_{k,j-1} + (j+1)\mu p_{k,j+1} + \lambda^* p_{k-1,j}$$

$$\Rightarrow p_{k,j+1} = \frac{j\mu + \lambda + k\mu^*}{(j+1)\mu} p_{k,j} \\ - \frac{\lambda}{(j+1)\mu} p_{k,j-1} - \frac{\lambda^*}{(j+1)\mu} p_{k-1,j}$$

or for $i = k, 1 < 0 \le c - k + 1$

$$p_{k,j} = \frac{(j-1)\mu + \lambda + k\mu^*}{j\mu} p_{k,j-1} - \frac{\lambda}{j\mu} p_{k,j-2} - \frac{\lambda^*}{j\mu} p_{k-1,j-1}$$
(8)

Case G. $0 < i < k, c - i \le j < c$ (see Figure 3 (G))

$$[j\mu + \lambda^* + \lambda + (c - j)\mu^*]p_{i,j}$$

$$= \lambda p_{i,j-1} + (c - j)\mu^* p_{i+1,j}$$

$$+ (j+1)\mu p_{i,j+1} + \lambda^* p_{i-1,j}$$

After re-arrangement,

$$p_{i,j+1} = \frac{j\mu + \lambda^* + \lambda + (c-j)\mu^*}{(j+1)\mu} p_{i,j}$$

$$- \frac{\lambda}{(j+1)\mu} p_{i,j-1} - \frac{\lambda^*}{(j+1)\mu} p_{i-1,j}$$

$$- \frac{(c-j)\mu^*}{(j+1)\mu} p_i + 1, j$$

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or for
$$0 < i < k, c - i + 1 \le j \le c$$

$$p_{i,j} = \frac{(j-1)\mu + \lambda^* + \lambda + (c-j+1)\mu^*}{j\mu} p_{i,j-1}$$

$$- \frac{\lambda}{j\mu} p_{i,j-2} - \frac{\lambda^*}{j\mu} p_{i-1,j-1}$$

$$- \frac{(c-j+1)\mu^*}{j\mu} p_{i+1,j-1}$$
(9)

Case H. $i = k, c - k \le j < c$ (see Figure 3 (H))

$$[j\mu + \lambda + (c - k)\mu^*]p_{k,j}$$

$$= \lambda p_{k,j-1} + (j+1)\mu p_{k,j+1} + \lambda^* p_{k-1,j}$$

$$\Rightarrow p_{k,j+1} = \frac{j\mu + \lambda + (c-k)\mu^*}{(j+1)\mu} p_{k,j}$$

$$-\frac{\lambda}{(j+1)\mu} p_{k,j-1} - \frac{\lambda^*}{(j+1)\mu} p_{k-1,j}$$

or for $i = k, c - k + 1 \le j \le c$

$$p_{k,j} = \frac{(j-1)\mu + \lambda + (c-k+1)\mu^*}{j\mu} p_{k,j-1} - \frac{\lambda}{j\mu} p_{k,j-2} - \frac{\lambda^*}{j\mu} p_{k-1,j-1}$$
(10)

Case I. For i = 0, j = c (see Figure 4 (I))

$$(c\mu + \lambda^*)p_{0,c} = \lambda p_{0,c-1} \Rightarrow p_{0,c} = \frac{\lambda}{c\mu + \lambda^*}p_{0,c-1}$$
(11)

Case J. For 0 < i < k, j = c (see Figure 4 (J))

$$(c\mu + \lambda^*)p_{i,c} = \lambda p_{i,c-1} + \lambda^* p_{i-1,c}$$

$$\Rightarrow p_{i,c} = \frac{\lambda}{c\mu + \lambda^*} p_{i,c-1} + \frac{\lambda}{c\mu + \lambda^*} p_{i-1,c}$$
(12)

Case K. For i = k, j = c (see Figure 4 (K))

$$c\mu p_{k,c} = \lambda p_{k,c-1} + \lambda^* p_{k-1,c}$$

$$\Rightarrow p_{k,c} = \frac{\lambda}{c\mu} p_{k,c-1} + \frac{\lambda^*}{c\mu} p_{k-1,c}$$
(13)

Equations (3)-(13) together with the equation

$$\sum_{0 \le j \le c, 0 \le i \le k} p_{i,j} = 1$$

solve to yield the $p_{i,j}$ values, and $p_b^{(n)}$ is computed as

$$p_b^{(n)} = \sum_{0 \le j \le c} p_{k,j}$$

for N' >> N (in our study, N' = 100N is selected)

$$p_b \simeq \sum_{1 \le n \le N'} \left[\pi_n p_b^{(n)} \right] \tag{14}$$

Several observations and extensions to this model are in order:

• The probabilities $p_{i,j}$ derived in this model hold for both the FIFO scheduling and the processor sharing scheduling [9].

In the FIFO scheduling (this policy may be adopted in CDPD if the circuit switching mode is provided in CDPD), every CDPD channel stream is assigned to exactly one M-ES. If k=4 and only two CDPD channel streams are available, then the third (and the fourth) M-ESs will be in the waiting queue. In the processor sharing scheduling, the four M-ESs will share the two streams in the round robin fashion.

- If the maximum number of the M-ESs allowed in the system is k' > k, then the state space should be modified such that $0 \le i \le k'$, and the rate from (i,j) to (i-1,j) is $i\mu^*$ (for $i \le \min(k',c-j)$) or $\min(k',c-j)\mu^*$ (for $i > \min(k',c-j)$).
- If a CDPD user can only issue one request at a time, then the state space should be modified such that $0 \le i \le \min(k, n)$, and the rate from (i, j) to (i + 1, j) should be $(n i)\overline{\lambda}$.

3 Discussions and Conclusions

Based on the availability model, we study the effects of the input parameters on the CDPD performance. In all cases studied in this section, the mean voice call holding time is 1 minute. Unless state otherwise, the number of channels in a cell is c=10, the number of the CDPD channel streams allowed in the system is k=2, the average net CDPD request rate is 1 request per minute $(\overline{\lambda^*}=\mu)$, the CDPD service time is 15 seconds $(\mu^*=4\mu)$, the Cellular user moves to a new cell every 10 minutes $(\eta=0.1\mu)$, and the voice call arrival rate range from 0.1-0.7 calls per minute per channel. Based on (14), we investigate the effect of the input parameters on the CDPD availability.

The effect of N. Figure 5 indicates that the ex-

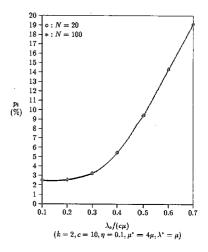


Figure 5: The effect of the expected number N of the M-ESs

pected number of the M-ESs only has insignificant effect on p_b . Note that the population distribution (see Equation (1)) of M-ESs are very different for N=20 and 100. We conclude that the M-ES distribution due to mobility does not have effect on the CDPD blocking probability. In the remainder of this section, N=50 is assumed.

The effect of the expected net CDPD

request rate $\overline{\lambda^*}$. Figure 6 (a) illustrates the effect of the expected net CDPD request rate $\overline{\lambda^*}$. The figure indicates that if $\overline{\lambda^*}$ is sufficient small (e.g., less than one request per two minutes or $\overline{\lambda^*} \leq 0.5\mu$), then different CDPD request rates do not change p_b significant. On the other hand, when $\overline{\lambda^*} \geq 0.5\mu$, increasing the CDPD request rate will significantly increase the blocking probability p_b .

The effect of the expected CDPD

completion rate μ^* . Figure 6 (b) illustrates the effect of the CDPD completion rate μ^* . It is obvious that the faster the completion rate μ^* the lower the blocking probability. We observe that when the background traffic is high, the effect of μ^* becomes less significant (especially when μ^* is

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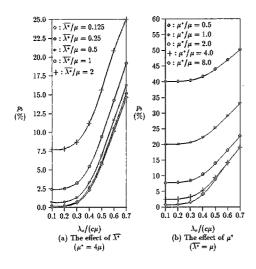


Figure 6: The effect of μ^* and $\overline{\lambda^*}$

small). However, if the CDPD traffic $\overline{\lambda^*}/\mu^*$ is a constant, we see the reverse result as shown in Figure 7. This phenomenon is explained as follows. For a constant CDPD traffic, faster completion rate implies larger CDPD request rate. Thus, in a short observed period, the system will experience more CDPD requests (with short service times). Since the number of CDPD channel streams is limited, it is more likely that more requests are blocked. If the background voice traffic is low, then more CDPD channel streams are available, and the large number of the requests can be absorbed by these CDPD channel streams, and the effect of large request rate disappears.

The effect of the Cellular user mobility η . High Cellular user mobility η results in a large voice handoff traffic, and thus a large voice traffic to a cell. Therefore, as indicated in Figure 8 (a), it is apparent that the CDPD blocking probability p_b increases as the Cellular user mobility increases.

The effect of the number of CDPD channel streams k. Figure 8 (b) indicates that for $k \geq 3$, increasing the number of the CDPD

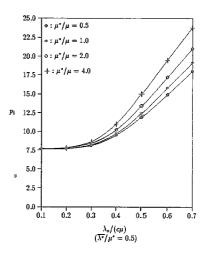


Figure 7: The effect of μ^* and $\overline{\lambda^*}(cont.)$

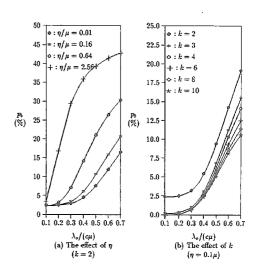


Figure 8: The effects of η and k.

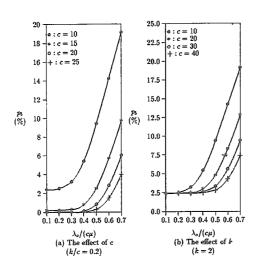


Figure 9: The effects of c.

channel streams only provide marginal benefit of the channel availability. Since adding CDPD channel streams will increase hardware complexity, it may not be cost-effective to increase the number of the streams for $k \geq 3$.

The effect of c. It is apparent that increasing the number of radio channels c will reduce the CDPD blocking probability (see Figure 9). Even if the number of CDPD channel streams is fixed, increasing the number of c still reduces p_b (see Figure 9 (b)). This benefit disappears when the voice traffic is small.

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