針對分散式計算系統
以馬可夫鏈結為基礎之可靠度分析
Markov-Chain-Based Reliability Analysis for Distributed Computing Systems

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摘要

本文提出一個新的模式來評估一個典型之分散式計算系統(DCS)的可靠度，並且提出二項可靠度評估方法：馬可夫鏈結分散式程式可靠度(MDPR)和馬可夫鏈結分散式系統可靠度(MDSR)，以便更加趨近於分散式計算系統的真實狀況．此外，並利用模擬的方式驗證其正確性.

關鍵字：分散式計算系統，分散式系統，可靠度，馬可夫鏈結.

Abstract

In this paper, a new model based on the DPR (distributed program reliability) and DSR (distributed system reliability) is presented to evaluate the reliability in a typical distributed computing system (DCS). Two reliability measures are introduced which are Markov-chain distributed program reliability (MDPR) and Markov-chain distributed system reliability (MDSR) to capture a more realistic view of the behavior of DCS in actual operating conditions. In addition, a simulation result is also presented to prove its correction.

Keywords: Distributed Computing System, Distributed Systems, Reliability, Markov-chain

1. Introduction

A typical DCS reliability problem is often modeling the network as a graph G(V,E), where the V is the set of nodes which denote processing elements and the E is the set of edges which denote communication links [1]. In this paper, we use distributed program reliability (DPR) and distributed system reliability (DSR) which is defined in [2] as a basis to evaluate the reliability of a DCS. In a DCS, programs and data files are distributed at several processing elements/nodes. For a successful execution of a program, these processing elements must cooperate in the execution of a program via communication links. In the meantime, the local processing element holding the executing program, the processing elements holding the required files, and the interconnecting links must all be operational. Distributed program reliability is the probability that a program can run successfully on a DCS. Distributed system reliability, which is the reliability measure of the entire system, is defined to be the probability that a set of given programs can run successfully on a DCS.

In the evaluation of DPR and DSR, the File Spanning Tree (FST) of the DCS must firstly be found. An FST which is a spanning tree that connects the root node (the processing element that runs the program under consideration) to some other nodes such that its vertices hold all the needed files for the program. A Minimal File Spanning Tree (MFST) is also a FST such that there is no other FST which is subset of it. Consequently, the DPR is the probability that at least one disjoint MFST or a given program is working, and DSR is the probability that at least one disjoint MFST of all the distributed programs are working.

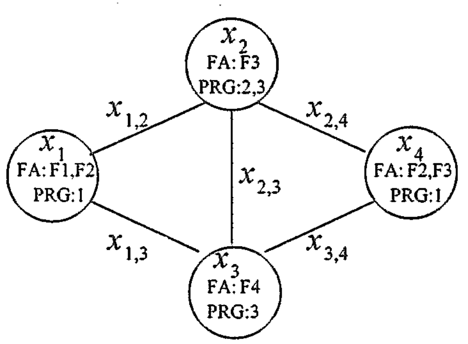
In the previous research, we have known that the computation of DPR and DSR depends not only on the system topology, but also on the distribution of programs and data-files[3,4].

Moreover, we model the system as a discrete time Markov model. The state space of Markov-chain is organized by the combinations of link up and down conditions. To improve the reliability, a repairable system which the links with finite repair rate is proposed to evaluate the impact for reliability, and different link failure rates will be taken to show how reliability change.

The rest of this paper is organized as follows. The system models are described in section 2. Section 3 is the model analysis which describes the analytical derivation and the procedures to evaluate the measures of MDPR, MDSR and MTTF. Section 4 illustrates the reliability analysis results and simulation results. Section 5 gives the conclusions.

2. System Models

Considering the network graph shown in figure 1, the distribution of programs, data-files and files required for program execution are listed. Each node is assumed to be perfectly reliable such that it never fails. Each link in the network may have two states, up (working) or down (failed) such that there are $2^{|E|}$ combinations of link states. The total state space size is therefore $2^{|E|}$. In the case of figure 1 there are total 32 states. If the failure of links are independent of each other, and the up/down combinations of the links construct a state. Thus, we can model the system with finite state space $\vartheta=\{0,1,2,\ldots,m\}$, whose stochastic behavior in time is described by a time-homogeneous Markov chain $X=\{X_t$, where t is the set of time points.



FA1={F1,F2}	PRG1={P1}	FN1={F1,F2,F3}
FA2={F3}	PRG2={P2,P3}	FN2={F1,F2,F4}
FA3={F4}	PRG3={P3}	FN3={F1,F2,F3,F4}
FA4={F2,F3}	PRG4={P1}	

Figure 1. A simple DCS with four nodes and five links

Assumptions

1. A DCS is modeled by a undirected graph G, where nodes denote processing elements, and edges denote communication links.
2. A link has 2 states : up and down.
3. Link failure rates are statistically independent and exponentially distributed.
4. Link repair rates are statistically independent and exponentially distributed.
5. Transitions from one state into another state involve link failures and link repairs.
6. At any unit of time, there is at most one link can fail or repair.
7. Each processing element is perfectly reliable.

Definition 1 : A File Spanning Graph (FSG) is a subgraph of the network, such that it contains at least a FST.

Definition 2 : Markov chain distributed program reliability (MDPR)

$$MDPR_i(t) = \quad - P_{r_F}\{t\}$$

where F is the failed state of program i

Definition 3 : Markov chain distributed system reliability (MDSR)

$$MDSR_{system}(t) = 1 - P_{r_F}\{t\}$$

where F is the failed state of all the executing programs in the system

Reliability analysis

Reliability assessment of a DCS can be classified by the following cases.

(1) Systems without repair capability :
A link functions for an exponentially distributed time with rate λ and then fails.

(2) Systems with repair capability :
Links have a constant failure rate λ with exponentially distributed. Once a link fail, it takes an exponentially distributed time with rate θ to be repaired.

3. Models Analysis

Assume that ϑ is the state space of the system with a finite number of states. Let the system be observed at the discrete moments of time t = 0,1,2...., and let X_t denote the state of the system at time t. Thus

$$\Pr(X_{t+1}=x_{t+1}|X_0=x_0,\ldots,X_t=x_t)=\Pr(X_{t+1}=x_{t+1}|X_t=x_t)$$

for every choice of the nonnegative integer t and the numbers $x_0,\ldots,x_{t+1}$, each in ϑ. Therefore it satisfies the Markov property. X_t denotes a m-state finite Markov chain, where t is in (0,1,2,.....,m-1). Let its transition probability matrix P and initial state-probabilities be given by

$$P=\begin{pmatrix} p^{00} & p^{01} & \cdots & p^{0,m-1} \\ p^{10} & p^{11} & \cdots & p^{1,m-1} \\ \vdots & \vdots & & \vdots \\ p^{m-1,0} & p^{m-1,1} & \cdots & p^{m-1,m-1} \end{pmatrix}$$

where $P_{ij}^{(s)} = \Pr(X_{k+s} = j | X_k = i)$, i,j = 0,1,....m-1; s≥1 and $P_{ij}^{(1)} \equiv P_{ij}$

Theorem 1 : Let P be the transition probability matrix of a finite Markov chain with elements P_{ij} (i,j=0,1,2,...,m-1). The n-step transition probabilities $P_{ij}^{(n)}$ are then obtained as the element of the matrix P^n. And we can regard P^n as an n-step transition matrix.

In general, the Markov chain constructed for any network contains one absorbing and m-1 transition states. In our model, all of the operational states are transient states. And all of the failed states are aggregated into a single absorbing state W. Rearranging the states of the system, the transition probability matrix P can be put in the following canonical form :

$$P = \begin{array}{c} \text{States} \\ W \\ 1 \\ 2 \\ \vdots \\ (m-1) \end{array} \begin{array}{c} W \quad 1 \quad 2 \ldots\ldots (m-1) \\ \left[\begin{array}{c|ccc} 1 & 0 & 0 \cdots & 0 \\ \hline & & & \\ R & & Q & \\ & & & \end{array}\right] \end{array}$$

where

Q: A (m-1) × (m-1) substochastic matrix with probabilities of transition only among the transient states for its elements;

R: A (m-1) × 1 matrix whose elements are the probabilities of the one-step transition form the (m-1) transient states to the r recurrent states.

Let $M = (I-Q)^{-1}$ be the fundamental matrix that is available in the derivation of N_i.

Theorem 2: Let $\mu_{ij} = E[N_{ij}]$, for i,j $\in T$

$$\|\mu_{ij}\| = M$$

Theorem 3: Let $M_\rho = \left\| \sum_{j \in T} \mu_{ij} \right\|$, then $\|E(N_i)\| = M_\rho$

In the next section, theorem 1 through theorem 3 will be helpful in evaluating MDPR, MDSR and MTTF.

We first take network graph shown in figure 1 as an example. For the convenience, link $x_{1,2}$, $x_{1,3}$, $x_{2,3}$, $x_{2,4}$, $x_{3,4}$ are labeled by e_1, e_2, e_3, e_4, e_5. Each state of the Markov chain represented by {ij...k}, for $e_i, e_j, \ldots, e_k$, denoting link $e_i, e_j, \ldots, e_k$ has failed. Their states are shown in figure 2.

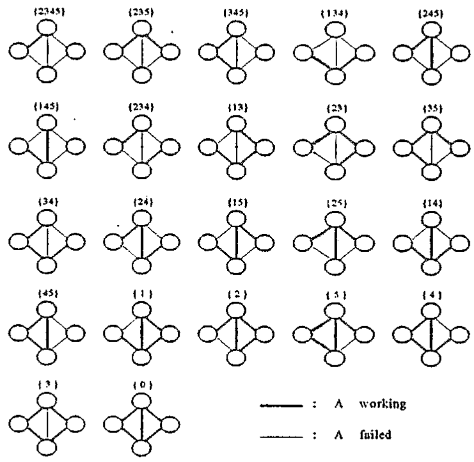


Figure 2. All the FSGs of program 1

According to the assumptions in section 2, the state transition diagram can be shown in figure 3, which presents the nonrepairable condition.

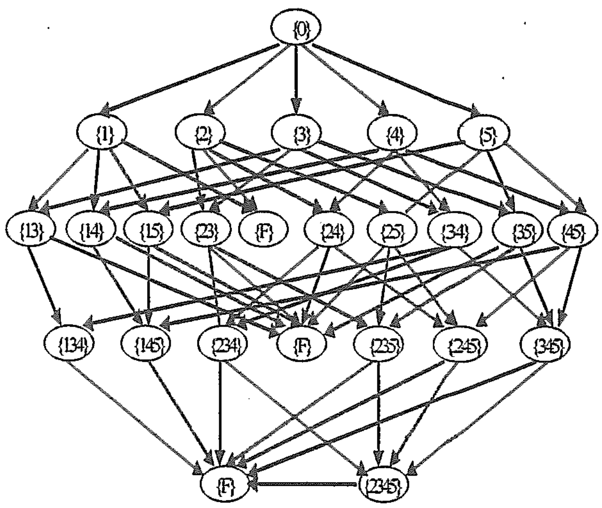


Figure 3. The Markov chain process of MDPR1

Transition from state S1 to S2 implies that there is a link fails at that time interval.

4. Analysis AND Simulation Results

Based on previous example, we try to evaluate MDPR1 and MTTF derived in theorem 1 through theorem 3.

System without repair capability

Table 1 list the detailed state transition condition of $MDPR$. Each link with a constant failure rate λ. For example, the failure of e_4 in state {23} → {234} with a failure rate λ is described by a state transition 9 → (17,λ). Notice that there exists 6 → (22,2λ), it is because a transition rate of $2\times\lambda$ since it is triggered by the failure of link e_2 or link e_5.

Table 1
MDPR1 MARkov Chain
[Without Repair Capability

State	Label	Transition
{0}	0	(1,λ),(2,λ),(3,λ),(4,λ),(5,λ)
{1}	1	(6,λ),(7,λ),(8,λ),(22,λ)
{2}	2	(9,λ),(10,λ),(11,λ),(22,λ)
{3}	3	(6,λ),(9,λ),(12,λ),(13,λ)
{4}	4	(7,λ),(10,λ),(12,λ),(14,λ)
{5}	5	(8,λ),(11,λ),(13,λ),(14,λ)
{13}	6	(15,λ),(22,2λ)
{14}	7	(16,λ),(22,2λ)
{15}	8	(16,λ),(22,2λ)
{23}	9	(17,λ),(18,λ),(22,λ)
{24}	10	(17,λ),(19,λ),(22,λ)
{25}	11	(18,λ),(19,λ),(22,λ)
{34}	12	(15,λ),(17,λ),(20,λ)
{35}	13	(18,λ),(20,λ),(22,λ)
{45}	14	(16,λ),(19,λ),(20,λ)
{134}	15	(22,2λ)
{145}	16	(22,2λ)
{234}	17	(21,λ),(22,λ)
{235}	18	(21,λ),(22,λ)
{245}	19	(21,λ),(22,λ)
{345}	20	(21,λ),(22,λ)
{2345}	21	(22,λ)
{F}	22	none

System with finite repair rate

Table 2 lists the detailed state transition condition under a constant repair rate θ. The failure rate condition is similar to table 1, except that a repair rate is considered.

Table 2
MDPR1 Markov
[Finite Repair-rate]

State	Label	Transition
{0}	0	(1,λ),(2,λ),(3,λ),(4,λ),(5,λ)
{1}	1	(6,λ),(7,λ),(8,λ),(22,λ),(0,θ)
{2}	2	(9,λ),(10,λ),(11,λ),(22,λ),(0,θ)
{3}	3	(6,λ),(9,λ),(12,λ),(13,λ),(0,θ)
{4}	4	(7,λ),(10,λ),(12,λ),(14,λ),(0,θ)
{5}	5	(8,λ),(11,λ),(13,λ),(14,λ),(0,θ)
{13}	6	(15,λ),(22,2λ),(1,θ),(3,θ)
{14}	7	(16,λ),(22,2λ),(1,θ),(4,θ)
{15}	8	(16,λ),(22,2λ),(1,θ),(5,θ)
{23}	9	(17,λ),(18,λ),(22,λ),(2,θ),(3,θ)
{24}	10	(17,λ),(19,λ),(22,λ),(2,θ),(4,θ)
{25}	11	(18,λ),(19,λ),(22,λ),(2,θ),(5,θ)
{34}	12	(15,λ),(17,λ),(20,λ),(3,θ),(4,θ)
{35}	13	(18,λ),(20,λ),(22,λ),(3,θ),(5,θ)
{45}	14	(16,λ),(19,λ),(20,λ),(4,θ),(5,θ)
{134}	15	(22,2λ),(6,θ),(12,θ)
{145}	16	(22,2λ),(7,θ),(8,θ),(14,θ)
{234}	17	(21,λ),(22,λ),(9,θ),(10,θ),(12,θ)
{235}	18	(21,λ),(22,λ),(9,θ),(11,θ),(13,θ)
{245}	19	(21,λ),(22,λ),(10,θ),(11,θ),(14,θ)
{345}	20	(21,λ),(22,λ),(12,θ),(13,θ),(14,θ)
{2345}	21	(22,λ),(17,θ),(18,θ),(19,θ),(20,θ)
{F}	22	none

We are not only interested in evaluating the MDPR1 under a constant λ, but also to realize the impact for different failure rate λ and repair rate θ. By using the contents of table 1, we can gain the transition probability matrix P and then Transform P to a canonical form. Hence, after t unit of time from initial state, $MDPR_1(t)$ is then obtained by $-\ P^t[i][j]$, where i is the initial state and j is absorbing state (In our model, it is a failed state). P[i][j] indicates the probability transition form state i to state j.

Figure 4 shows the MDPR1 under different link failure rate. Figure 5 shows that under a constant link failure rate 0.03, the impact of MDPR1 for different link repair rates. Compare to Figure 4 and Figure 5, we can conclude that the reliability of distributed program 1 can benefit more from an decrease of the link failure rate then the increase of link repair rate. In fact, the reliability of distributed program 1 becomes better than that of a higher link repair rate when λ below a threshold value. Figure 6 and Figure 7 is the simulation result at the same condition, it also supports this view of point.

Mean time to failure (MTTF)

Mean time to failure is defined as the total number of times spend in transient (operational) state before entering a absorbing (failed) state. Hence, it is the measure $\|E(N_i)\|$ described in theorem 3. Figure 8 and Figure 9 shows the value of MTTF for different link failure rate from initial state.

Choice of link

Referring to figure 10, it provides us another view of figure 4. Its availability is that, for example, what is the answer of link reliability under system lifetime ≥ T and subject to a program/system reliability constraint.

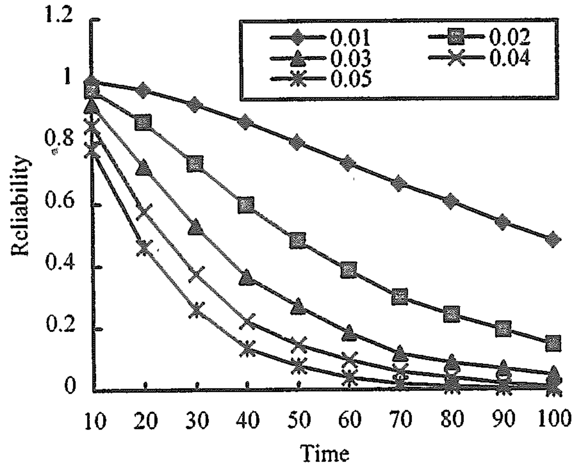


Figure 4. Mathematical analysis on different failure rates

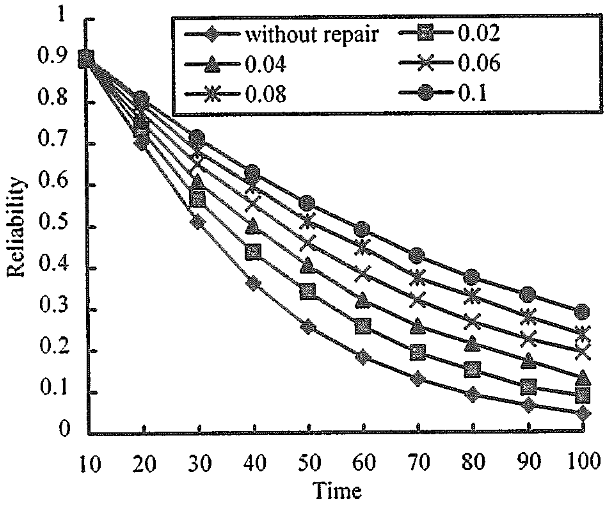


Figure 5. Mathematical analysis on different repair rates

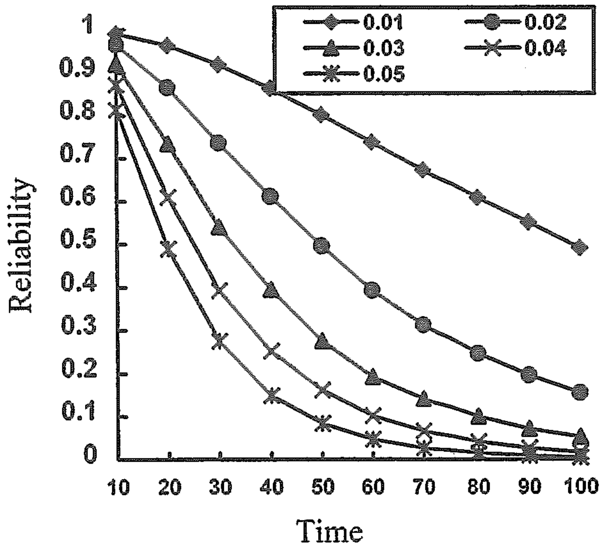


Figure 6. Simulation analysis on different failure rate

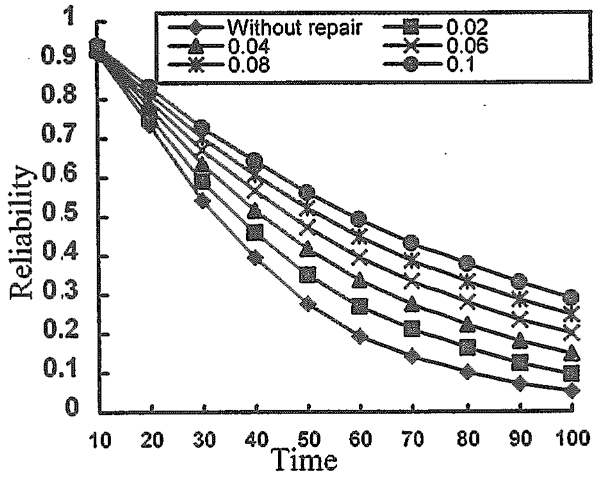


Figure 7. Simulation analysis on different repair rate

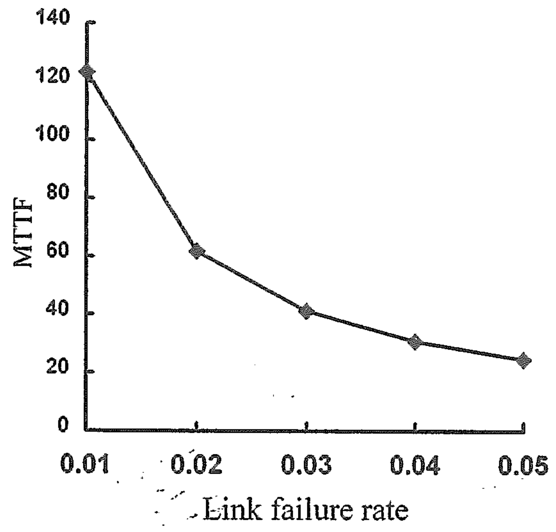


Figure 8. The comparison of MTTF under different link failure rate

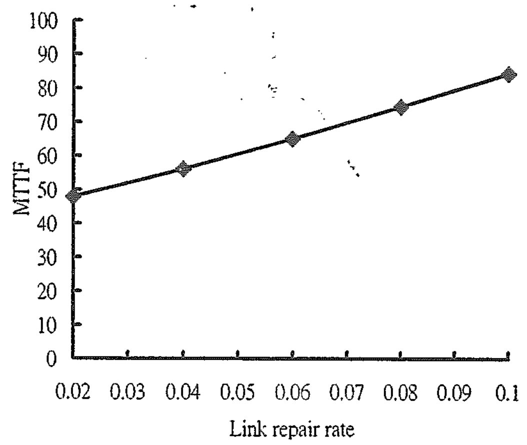


Figure 9. The comparison of MTTF under constant $\lambda = 0.03$ and different link repair rate

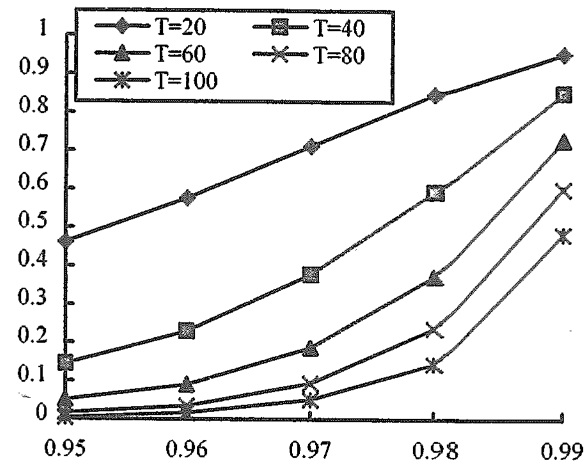


Figure 10. MDPR1 versus link reliability on different time points

5. Conclusions

Many researches have addressed on the study of reliability analysis on distributed systems or networks. In this paper, a new model has presented to meet Markov chain property. The failure/repair rates of each link is independent and exponentially distributed. Thus, the "time" behavior of the system is described by Markov chain $X = \{X_t;\ t = 0,1,2,.....\}$ with a given transition matrix and initial state-probabilities. By means of a stochastic model based on a Markov chain with a absorbing state, the system reliability can then be described as a function of time, such that it can capture a more realistic behavior. This paper consider link up/down combinations as the system states to simplify the model and the evaluation. From the simulation, the result of the analysis model is quite similar to that of the simulation. The system with the capacity of repairment can have higher reliability for all situations

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