THE DESIGN OF TWO-DEGREE-OF-FREEDOM MULTIVARIABLE CONTROL SYSTEMS

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Abstract

A three-element multivariable control system with two-degree-of-freedom (2-df) is proposed. Among the three elements, one in the main loop is designed as an inverse-based controller for rejecting disturbance, and the other two which serve as pre-filter and dynamic preset are devised for set-point tracking. These elements can be designed to satisfy two desired objectives independently and with emphasis on their physical realization. The Simulation results show that satisfactory control performance can be achieved.

Keywords: Multivariable control, Decoupling, Two-degree-of-freedom control, Prediction, Non-interacting

1. Introduction

In general, most of chemical plants are MIMO systems. With increasing competition in market, the manufacturers are forced to have higher product quality with lower cost, and thus process naturally need tighter control. In the past, multi-loop SISO controllers are practically used to control a MIMO system. However, designs of controllers for such a system (Luyben, 1986; Shen and Yu, 1994)[1], [2] are usually coupled due to the interactions. To overcome the difficulties in design, Huang (2003) [3] decomposed the multi-loop system into a number of equivalent single loops for design. But this type of multi-loop SISO controllers usually brings interactions to other loops. Theoretically, by multivariable controllers, these interactions can be eliminated as much as possible. Wang et al. (1997) [4] proposed the fully cross-coupled multivariable PID controllers and they (2003) [5], [6] also proposed a method to design general multivariable controller. In their method, the desired objective loop transfer functions are targeted to obtain good performances for step set-point changes. But, the resulting system has sluggish responses for disturbance input. In addition, the objective closed-loop transfer functions are specified by some complex procedures.

In literatures, a 2-df control or relevant structures can achieve two objectives simultaneously within one system. Nevertheless, the two controllers are highly dependent in design. Tian and Gao (1998) [7] proposed a double-controller scheme, a set-point controller and a load controller. In their control structure, the two controllers can be designed independently to achieve good system performance for both set-point tracking and load rejection. But, direct using their structure to control multivariable systems will cause some problems. For examples, the resulting control system is complex and the dead-time can't compensate completely in loops. In this paper, motivated by the double-controller of Tien and Gao aforementioned, a new 2-df multivariable control system with three controller elements is presented. One of the elements as an inverse-based controller in devised in the main loop for rejecting disturbance. The design of this inverse-based controller emphasizes on a systematic procedure to obtain physically realizable controllers for practical implementation. It can reject the disturbance effectively under the desired robustness. The other two elements (one as pre-filter and one as dynamic preset), are designed to have a dead-time compensated response as that of a Smith predictor. The two elements mentioned have explicit functional relations to the desired control specification and the open-loop dynamics. Thus, they can be synthesized very easily.

By making use of this 2-df system, simulations using example processes have been applied to. The results on both of set-point tracking and disturbance rejection performances are satisfactory.

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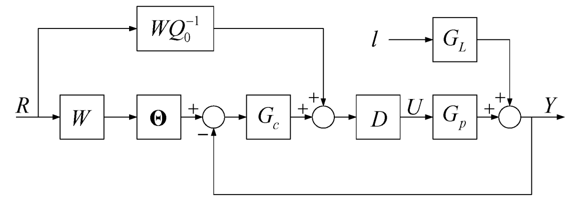

Fig. 1. A 2-df multivariable control structure

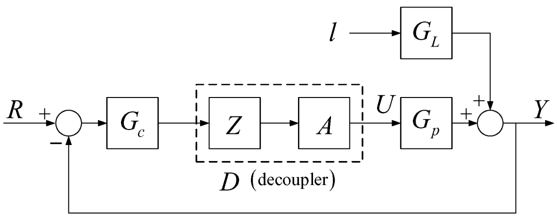

Fig. 2. A multivariable decoupling control system

2. Two-degree-of-freedom multivariable control structure

Consider a m×m system of the following:

$$Y(s) = G_P(s)U(s) + G_L(s)L(s) \tag{1}$$

where $Y(s)$ and $U(s)$ designate the output and input vector, respectively. $L(s)$ and $G_L(s)$ are the disturbance and the dynamics from disturbance to each loop. $G_p(s)$ is an open-loop transfer function matrix (abbr. TFM) that represents the dynamics of the plant. In general, $G_p(s)$ and $G_L(s)$ are given as:

$$G_p(s) = \begin{bmatrix} g_{11}(s) & \cdots & g_{1m}(s) \\ \vdots & \ddots & \vdots \\ g_{m1}(s) & \cdots & g_{mm}(s) \end{bmatrix} \tag{2}$$

and

$$G_L(s) = \begin{bmatrix} g_{L1}(s) \\ \vdots \\ g_{Lm}(s) \end{bmatrix} \tag{3}$$

where $g_{ij}(s)$ are strictly proper, stable scalar rational functions. To simplify the problem, the open-loop TFM in Eq. (2) is rewritten as:

$$G_p(s) = \begin{bmatrix} e^{-\theta_1 s} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & e^{-\theta_m s} \end{bmatrix} G_{p0}(s) = e^{-\mathbf{\theta} s} G_{p0}(s) \tag{4}$$

where $\mathbf{\theta}$ is a diagonal matrix composing of each θ_i (i=1,2,…,n) which is the shortest delay time in each i-th row of $G_p(s)$. With the process transfer function matrices given above, the 2-df control system with three elements proposed is as shown in Fig. 1.

2.1 Design of the inverse-based controller

In Fig. 2, the inverse-based controller consists of three components: $G_c(s)$, $A(s)$, and $Z(s)$ with the definitions as follows:

$$G_c(s) = Diag\left\{g_{c,i}(s);\ i = 1,2,\ldots,m\right\}; \tag{5}$$

$$A(s) = adj\left\{G_{p0}(s)\right\}; \tag{6}$$

and

$$Z(s) = Diag\left\{z_i(s);\ i = 1,2,\ldots,m\right\} \tag{7}$$

The model of $\det\{G_{po}(s)\}$ is divided into the delay free term and delay term, and that is:

$$\det\left\{G_{p0}(s)\right\} = \left[\det\left\{G_{p0}(s)\right\}\right]_0 e^{-\theta^* s} \tag{8}$$

where, θ^* and $[\det\{G_{p0}(s)\}]_0$ are the dead time and the other terms of the model of $\det\{G_{p0}(s)\}$. Furthermore, let us define:

$$Q_0(s) = Diag\left\{\left[\det\left\{G_{p0}(s)\right\}\right]_0 z_i(s);\ i = 1,2,\ldots,n\right\} \tag{9}$$

$$D(s) = A(s)Z(s) \tag{10}$$

Notice that $z_i(s)$ is a rational function to make $Z(s)A(s)$ is proper. By the definitions given above, it is obvious to have the following result:

$$\begin{aligned} G_p(s)A(s)Z(s) &= e^{-\left(\mathbf{\theta} + \theta^* I_m\right)s} Q_0(s) \\ &= Diag\left\{e^{-(\theta_i + \theta^*)s}; i = 1,\ldots n\right\} Diag\left\{q_{0i}(s); i = 1,\ldots,n\right\} \end{aligned} \tag{11}$$

Thus, the multivariable control system is thus decoupled into m individual SISO parallel loops. Noticed $Z(s)$ is used to re-allocate the poles of $[\det\{G_{po}(s)\}]_0$ and make all elements of $D(s)$ proper and physical realizable. By eliminating and re-allocating the poles, the order of transfer functions in $G_p(s)D(s)$ is reduced and the system can have faster response. The closed-loop responses to the set-point and to the load input, respectively, are then given as:

$$\begin{aligned} Y(s) = &\Big\{\left(I + G_p(s)D(s)G_c(s)\right)^{-1} G_p(s)D(s)G_c(s)\mathbf{\Theta}(s)W(s) \\ &+ \left(I + G_p(s)D(s)G_c(s)\right)^{-1} G_p(s)D(s)Q^{0-1}(s)W(s)\Big\} R(s) \\ = &W(s)\mathbf{\Theta}(s)R(s) \end{aligned} \tag{12}$$

and

$$Y(s) = \left(I + G_p(s)D(s)G_c(s)\right)^{-1} G_L(s)L(s) \tag{13}$$

From Eq. (12) and Eq. (13), it is easy to find that the responses of the system to the set-point and to the disturbance are decoupled completely through design of $W(s)$ and $G_c(s)$. Since the system is decoupled, multiple SISO controllers ($g_{ci}(s)$, $i = 1,\cdots,m$) can be designed to control the m decoupled single loop systems independently for disturbance rejection.

Design of $G_c(s)$. For each diagonal element of $G_p(s)D(s)$ (i.e. $e^{-(\theta_i + \theta^*)s}[\det\{G_{p0}(s)\}]_0 z_i(s)$), a SISO controller (i.e. $g_{ci}(s)$) is design. It is thus desirable to find an approximated model for $\det\{G_{p0}(s)\}$ first. In general, this transfer function can be described by the form of the following:

$$\det\left\{G_{p0}(s)\right\} \approx \frac{k e^{-\theta^* s} \prod_{i=1}^{n}\left(\tau_{L,i}s + 1\right)}{\left(as^2 + bs + 1\right)\prod_{i=1}^{p}\left(\tau_{g,i}s + 1\right)} \tag{14}$$

where n and p are the number of lag terms and lead terms, respectively. k, θ^*, $\tau_{g,i}$, $\tau_{L,i}$, a and b are the parameters of the approximated model $\det\{G_{p0}(s)\}$. In general, $n-p+2>0$ and has no RHP pole. To find $\det\{G_{po}(s)\}$, the following optimization problem is formulated. That is:

$$Arg\{\mathbf{P}\} = \min_{\mathbf{P}} \int_0^{\omega_f} \Big\{ W_a \left(\left\| G_{p0}(j\omega) \right| - \phi(j\omega) \right| \right)^2 + W_p \left(\angle G_{p0}(j\omega) - \angle \phi(j\omega) \right)^2 \Big\} d\omega \tag{15}$$

where $\phi(j\omega)$ is the model of $\det\{G_{p0}(s)\}$ and $\mathbf{p}$ consists of parameters in $\phi(j\omega)$. ω_f is the frequency bandwidth concerned and, W_a and W_p are the weight functions for the errors of magnitude ratio and phase angle, respectively. The parameters of the model can be obtained by minimum the objective function in Eq. (15). The order of numerator of $z_i(s)$ minus the order of denominator of $z_i(s)$ is defined as $O(z_i)$, which is chosen to make all elements of $A_{\bullet i}(s)z_i(s)$ proper. $A_{\bullet i}(s)$ means i-th column of $A(s)$.

Let:

$$\mathbf{\Theta}(s) = \begin{bmatrix} e^{-d_1 s} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & e^{-d_n s} \end{bmatrix} \tag{16}$$

where $d_i = \theta_i + \theta^*$. If the decoupling is perfect, the closed-loop transfer functions between the disturbance $l(s)$ and the process variable $Y(s)$ can be rewritten as:

$$y_i(s) = \frac{g_{Li}(s)}{1 + g_{ci}(s) q_{0i}(s) e^{-d_i s}} l(s), \quad \forall i \in [1, m] \tag{17}$$

Having these $q_{0i}(s)$ and $g_{Li}(s)$, the $g_{ci}(s)$ will be designed for rejecting the load disturbance. For a step and scalar load disturbance, $g_{ci}(s)$ is determined by minimizing the integral of the absolute value of the error, IAE. After we obtained the optimal PID controllers, $G_c(s)$, the loop transfer functions of main loops can be written as:

$$H(s) = G_p(s) D(s) G_{c2}(s) \tag{18}$$

Since $G_c(s)$ is designed based on the approximated model of $\det\{G_p^o(s)\}$, it is necessary to consider detuning the controller to ensure the robustness of stability. A detune factor, λ_i, is thus given to detune the controller gain of $g_{ci}(s)$ to give each loop has a proper gain margin. Thus, λ_i is found by:

$$\lambda_i = \left\{ \lambda_i \middle| GM\left(H_{ii}(s)\right) = 2 \right\} \quad \forall i \in [1, m] \tag{19}$$

Then, $G_c(s)$ can be obtained by the above design steps.

2.2 Design of two-element controller for set-point tracking

As mentioned, the inverse-based controller in the main loop decouples the open-loop process into SISO processes, that is:

$$G_p(s) A(s) Z(s) = G_p(s) D(s) = Q_0(s) \mathbf{\Theta}(s) \tag{20}$$

and

$$Y(s) = W(s) \mathbf{\Theta}(s) R(s) \tag{21}$$

From Eq. (21), we can specify $W(s)$ for set-point following performance, that is $diag\{w_i,\ i \in [1, m]\}$. The desired set-point following trajectory is closely related to decoupling results. In order to be practically realizable $w_i(s)/q_{0i}(s)$ must be proper and stable. The ith closed-loop response for set-point tracking can be expressed in terms of the following closed-loop transfer function:

$$w_i(s) = q_{0i}^+(s) \frac{1}{\left(\tau_{d1} s + 1\right)^n \left(\tau_{d2}^2 s^s + 2\tau_{d2} \zeta s + 1\right)} \tag{22}$$

where τ_{di} and ζ are the time constant and damping coefficient of our model. $q_{0i}^+(s)$ is the non-minimum phase zeros of $q_{0i}(s)$. Thus, there are no RHP poles in $w_i(s)/q_{0i}(s)$. Notice that, the decoupler is synthesized as the product of $A(s)$ and $Z(s)$. As an adjoint matrix of $G_{p0}(s)$, each element of $A(s)$ consists of multiplication and summation of fractional functions of s. Practically, when dimension of $G_p(s)$ is higher than 2, it would be easier to implement such elements with a simple form of function like Eq. (14). Similar situation happens to $adj\{G_{p0}(s)\}$. In the other words, elements of $A(s)$ and $\det\{G_{p0}(s)\}$ need to be approximated by Eq. (14). For this, modeling error may be introduced. If $A(s)$ are implemented with approximations, perfect decoupling may not always be possible. In that case, the $G_p(s)\bar{D}(s)$ will not be exactly diagonal, where, $\bar{D}(s)$ is the decoupler with approximation. An analogous closed-loop TFM from $R(s)$ to $Y(s)$ is thus defined as:

$$T(s) = G_p(s) \bar{D}(s) \left(Q_0(s)\right)^{-1} W(s) \tag{23}$$

Based on $T(s)$, a decoupling performance index is defined:

$$D_{ji}^p = \max_{\omega} \left(\frac{\left|T_{ji}(j\omega)\right|}{\left|T_{ii}(j\omega)\right|},\ i \neq j \right), \quad \forall \omega \in \left(0, \omega_{gi}\right] \tag{24}$$

where, ω_{gi} is the frequency bandwidth concerned and is taken as:

$$\omega_{gi} = \left\{ \omega \middle\| T_{ii}(j\omega) \right| = 0.707 \right\} \tag{25}$$

D_{ji}^p is the maximum magnitude ratio of the diagonal element, $T_{ii}(s)$, to the off-diagonal element, $T_{ji}(s)$, for $\omega < \omega_{gi}$. For robustness, it is recommend to assign each $w_i(s)$ to have $D_{ji}^p < \varepsilon_{pi} \in [0.1, 0.3]$. In other words, $w_i(s)$ is selected to satisfy:

$$w_i(s) = \left\{ w_i(s) \middle| D_{ji}^p < \varepsilon_{pi} \quad \forall j \in [1, m] \ and\ i \neq j \right\} \quad \forall i \in [1, m] \tag{26}$$

As above mentioned, the desired set-point response must be selected to satisfy the decoupling performance. Notice that the smaller value of ε_{pi} corresponds to the more stringent decoupling performance, but the loop performance will be more conservative. Similarly, modeling error exists due to approximation for $\det\{G_{p0}(s)\}$ so that $Q_0(s)\boldsymbol{\Theta}(\mathbf{s})$ will not exactly equal to $G_p(s)\bar{D}(s)$. An index, ε_{di}, to measure the discrepancy between the desired transfer function and the actual transfer function for set-point tracking is defined as follows:

$$\varepsilon_{di} = \max_{\omega}\left\{\frac{\left| T_{ii}^{d}(j\omega) - T_{ii}^{r}(j\omega)\right|}{\left|T_{ii}^{r}(j\omega)\right|},\ \omega\in\left[0,\omega_{gi}\right]\right\} \quad (27)$$

where

$$T_{ii}^{d}(s) = w_i(s)e^{-d_i s};$$
$$T_{ii}^{r}(s) = \left[G_p(s)\bar{D}(s)W(s)Q_0^{-1}(s)\right]_{ii}; \quad (28)$$
$$\omega_{gi} = \left\{\omega \,\middle|\, \left|T_{ii}^{r}(j\omega)\right| = 0.707\right\}$$

The value of ε_{di} can be reduced by increasing the order of the approximation model with the cost to increase the complexity of the control system. When the value of ε_{di} is large, the design of the system must be more conservative (e.g. increase the gain margin).

2.3 Overall controllers design procedure

Based on the theory given above, a systematic design procedure is summarized in the following. Give a stable m×m multivariable process $G_p(s)$ and ε_{di}.

(i) Partition $G_p(s)$ into two parts, that is:

$$G_p(s) = \begin{bmatrix} e^{-\theta_1 s} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & e^{-\theta_m s} \end{bmatrix} G_{p0}(s) \quad (29)$$

(ii) Get the approximated model of $\det\{G_{p0}(s)\}$.

(iii) Determine $A(s)$ by $adj\{G_{p0}(s)\}$. If the process is more than 2×2 system, approximated model of $adj\{G_{p0}(s)\}$ may be required for easy implementation.

(iv) According to the order of $A_{\bullet i}$, determine $O(z_i)$. Notice that $z_i(s)$ is used to cancel the sluggish poles of $\det\{G_{p0}(s)\}$ to accelerate the dynamic of $\det\{G_{p0}(s)\}$. Then, We can obtain decoupler by $D(s) = A(s)Z(s)$.

(v) Design the controller $G_c(s)$.

(vi) Choose $W(s)$ by Eq. (22) to satisfy the condition of Eq. (26) and determine $W(s)\left(Q_0(s)\right)^{-1}$.

(vii) Calculate the error of Eq. (27). Determine the desired gain margin and detune the controller gain of $G_c(s)$ by Eq. (19).

3. ILLUSTRATIVE EXAMPLES

3.1 Example 1

Consider the Ogunnaike and Ray (1994) [8] 2×2 process. The transfer function matrices of this process are given as follows:

$$G_p(s) = \begin{bmatrix} \dfrac{22.89e^{-0.2s}}{4.572s+1} & \dfrac{-11.64e^{-0.4s}}{1.807s+1} \\ \dfrac{4.689e^{-0.2s}}{2.174s+1} & \dfrac{5.8e^{-0.4s}}{1.801s+1} \end{bmatrix}; \quad G_L(s) = \begin{bmatrix} \dfrac{-4.243e^{-0.4s}}{3.445s+1} \\ \dfrac{-0.601e^{-0.4s}}{1.982s+1} \end{bmatrix} \quad (30)$$

First, partition $G_p(s)$ into two parts:

$$G_{p0}(s) = \begin{bmatrix} \dfrac{22.89}{4.572s+1} & \dfrac{-11.64e^{-0.2s}}{1.807s+1} \\ \dfrac{4.689}{2.174s+1} & \dfrac{5.8e^{-0.2s}}{1.801s+1} \end{bmatrix}; \quad e^{-\boldsymbol{\theta}s} = \begin{bmatrix} e^{-0.2s} & 0 \\ 0 & e^{-0.2s} \end{bmatrix} \quad (31)$$

The approximated model of $\det\{G_{p0}(s)\}$ is:

$$\det\{G_{po}(s)\} \approx \frac{187.34e^{-0.2s}}{(4.2243s+1)(1.4026s+1)} \quad (32)$$

$A(s)$ can be obtained from the adjoint of $G_{p0}(s)$. That is:

$$A(s) = \begin{bmatrix} \dfrac{5.8e^{-0.2s}}{1.801s+1} & \dfrac{11.64e^{-0.2s}}{1.807s+1} \\ \dfrac{-4.689}{2.174s+1} & \dfrac{22.89}{4.572s+1} \end{bmatrix} \quad (33)$$

Each element of $A(s)$ has one excess pole, thus $O(z_1(s)) = O(z_2(s)) = 1$. Let

$$z_1(s) = z_2(s) = (1.4026s+1) \quad (34)$$

Then, the decoupler can be given as $A(s)Z(s)$, that is:

$$D = \begin{bmatrix} \dfrac{5.8(1.4026s+1)e^{-0.2s}}{1.801s+1} & \dfrac{11.64(1.4026s+1)e^{-0.2s}}{1.807s+1} \\ \dfrac{-4.689(1.4026s+1)}{2.174s+1} & \dfrac{22.89(1.4026s+1)}{4.572s+1} \end{bmatrix} \quad (35)$$

Hence, we have $Q_0(s)$ as:

$$Q_0(s) \approx \begin{bmatrix} \dfrac{187.34}{(4.2243s+1)} & 0 \\ 0 & \dfrac{187.34}{(4.2243s+1)} \end{bmatrix} \quad (36)$$

We select the desired closed-loop transfer function, $W(s)$.

$$W(s) = \begin{bmatrix} \dfrac{1}{0.1s+1} & 0 \\ 0 & \dfrac{1}{0.1s+1} \end{bmatrix} \quad (37)$$

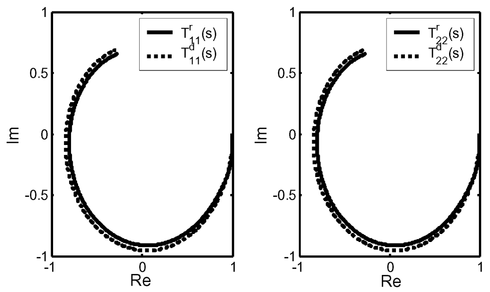

Fig. 3. The comparison between the real set-point response and desired set-point response.

Then, $W(s)Q_0^{-1}(s)$ and $\mathbf{\Theta}(s)$ can be obtained as:

$$W(s)Q_0^{-1}(s)=\begin{bmatrix}\dfrac{4.2243s+1}{0.1s+1} & 0\\ 0 & \dfrac{4.2243s+1}{0.1s+1}\end{bmatrix} \quad (38)$$

and

$$\mathbf{\Theta}(s)=\begin{bmatrix}e^{-0.4s} & 0\\ 0 & e^{-0.4s}\end{bmatrix} \quad (39)$$

The comparison between the real set-point response and desired set-point response is shown in Fig. (3). Because the modeling errors for the decoupler are small, a $GM=2$ is assigned. By the minimum IAE criterion, the optimal PID controllers are:

$$g_{c1}(s)=0.046\left(1+\frac{1}{0.50s}\right)\left(\frac{0.30s+1}{0.015s+1}\right)$$
$$g_{c2}(s)=0.047\left(1+\frac{1}{0.51s}\right)\left(\frac{0.31s+1}{0.015s+1}\right) \quad (40)$$

At GM=2, the detune factors are found to be:

$$\lambda_1=1.5;\ \lambda_2=1.52 \quad (41)$$

The simulation results for unit step set-point change and unit step disturbance input are given in Fig. 4 and Fig. 5 respectively. The results compare with the results of multi-loop controllers (Huang, 2003). In Fig. 4 and Fig. 5, we can find that the performances for set-point tracking and load rejection of our design are superior to the performances of multi-loop controllers. In addition, the interaction exists between the control loops in multi-loop control system. However, the proposed 2-df multivariable control structure results in satisfactory responses without loop interactions.

3.2 Example 2

The Tyreus (1982) [9] 3x3 process as follows is considered.

$$G_p(s)=$$
$$\begin{bmatrix}\dfrac{1.986e^{-0.71s}}{66.7s+1} & \dfrac{-5.24e^{-60s}}{400s+1} & \dfrac{-5.984e^{-2.24s}}{14.29s+1}\\ \dfrac{-0.0204e^{-0.59s}}{(7.14s+1)^2} & \dfrac{0.33e^{-0.68s}}{(2.38s+1)^2} & \dfrac{-2.38e^{-0.42s}}{(1.43s+1)^2}\\ \dfrac{-0.374e^{-7.75s}}{22.22s+1} & \dfrac{11.3e^{-3.79s}}{(21.74s+1)^2} & \dfrac{9.811e^{-1.59s}}{11.36s+1}\end{bmatrix} \quad (42)$$

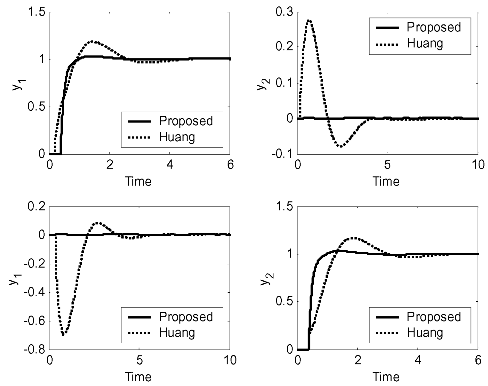

Fig. 4. Set-point tracking responses for OR (2x2) process.

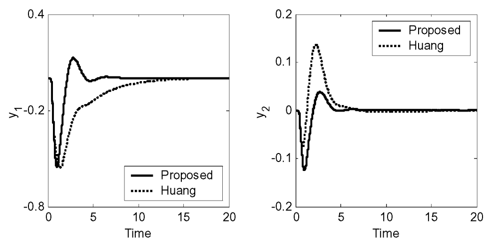

Fig. 5. Load rejecting responses for OR (2x2) process.

Assume the dynamic of disturbance is equal to the first column of $G_p(s)$, and that is:

$$G_L(s)=\begin{bmatrix}\dfrac{1.986e^{-0.71s}}{66.7s+1}\\ \dfrac{-0.0204e^{-0.59s}}{(7.14s+1)^2}\\ \dfrac{-0.374e^{-7.75s}}{22.22s+1}\end{bmatrix} \quad (43)$$

First, we find the approximated models of $\det\{G_{p0}(s)\}$ and $adj\{G_{p0}(s)\}$. By following the design procedure, we can obtain $A(s)$ and $Z(s)$, and calculate $D(s)$. Than, the desired decoupling performances are defined as

$$\varepsilon_{p1}=0.1;\ \varepsilon_{p2}=0.2;\ \varepsilon_{p3}=0.16 \quad (44)$$

To satisfy the Eq. (26), we select the desired set-point response as:

$$w_1(s)=\frac{1}{5s+1};$$
$$w_2(s)=\frac{1}{8^2s^2+2\times 8\times 0.7s+1}; \quad (45)$$
$$w_3(s)=\frac{1}{7s+1};$$

Then, design the optimal PID controllers for load rejection.

$$g_{c1}(s)=0.60\left(1+\frac{1}{1.14s}\right)\left(\frac{0.76s+1}{0.04s+1}\right)$$
$$g_{c2}(s)=24.15\left(1+\frac{1}{87.3s}\right)\left(\frac{1.58s+1}{0.08s+1}\right) \quad (46)$$
$$g_{c3}(s)=0.32\left(1+\frac{1}{2.27s}\right)\left(\frac{1.47s+1}{0.07s+1}\right)$$

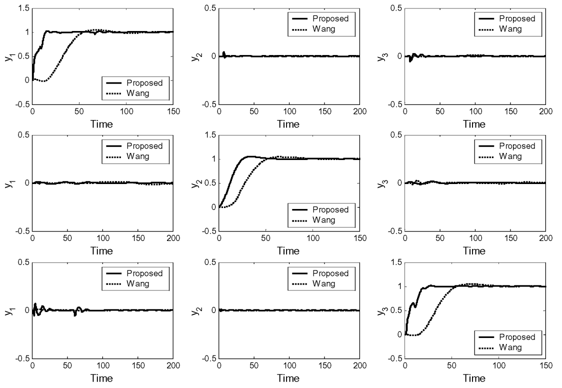

Fig. 6. Set-point tracking responses for Tyreus (3×3) process.

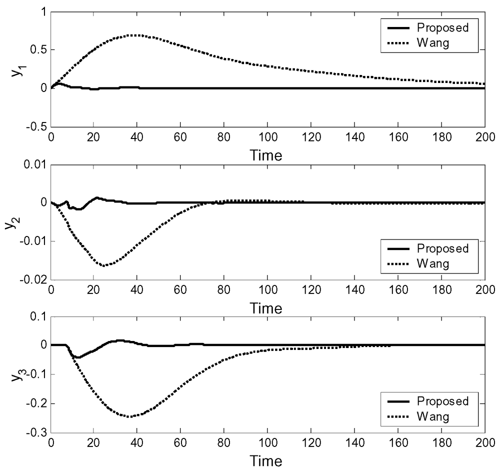

Fig. 7. Load rejecting responses for Tyreus (3×3) process.

At GM=3, use Eq. (19) to determine the detune factors:

$$\lambda_1 = 6.9;\ \ \lambda_2 = 5.4;\ \ \lambda_3 = 4.5 \qquad (47)$$

Simulation results for unit step change in set-point are given in Fig. 6. The results show that performances are compatible to the other reported design or even better. And the responses for load rejection are shown in Fig. 7. It is found that the conventional multivariable control system caused the sluggish responses for load rejection. In the proposed control system, it can get good performances for both servo tracking and load rejection. And the acceptable decoupling results are still maintained.

4. CONCLUSIONS

In this paper, a 2-df multivariable control structure has been proposed. It is easy applied to deal with both problems in servo tracking and load rejection. The method of decoupling loop interactions is based on fundamental linear algebra. For load rejection, the multivariable controllers is designed to eliminate the disturbance input with enough system robustness and the non-minimum phase poles are avoided directly in these controllers to guarantee the physical realization. The achievable set-point tracking response is also defined under the desired performance of loop decoupling, and it is easy applied in the proposed control structure. Furthermore, the response of set-point tracking is similar with the results of predictive control system. Not only two objectives can be achieved simultaneously while maintaining the minimum loop interactions and desired system robustness, but also the design of them are separable. Examples have illustrated that our approach can be achieved these objectives simultaneously.

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