An On-Line Control System Monitoring and Identification Using the Pulse Testing

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Abstract

A monitoring procedure using the pulse testing to calculate the maximum closed-loop log modulus ($L_{c,max}$) is presented in this study. Under an operation of closed-loop control, an arbitrary pulse in the set-point is introduced to the system. The closed-loop transients can be numerically translated into frequency response data by the Fourier integral transforms, and the $L_{c,max}$ can easily be calculated using these data. This technique has also been extended to the calculation of $L_{c,max}$ for $n \times n$ multivariable systems using n pulse tests. In addition, the parametric identification for given process models can also be obtained by a least-squares fit in frequency domain using the same experimental data. Simulation results have demonstrated that the proposed technique can yield a reliable $L_{c,max}$ rather easily as compared to those available methods in the literature.

1. Introduction

The maximum closed-loop log modulus, L_{c,max}, is considered to be a good measure of performance or robustness for a control system (Luyben, 1990). Chiang and Yu (1993) proposed an on-line frequency domain monitoring procedure to identify L_{c,max}, using relay feedback experiments for single-inputsingle-output (SISO) systems. In their work, two to three relay experiments is required in order to obtain the L_{c,max}, for a SISO system. Extension of this work to the calculation of L_{c,max}, using relay experiments for SISO and multivariable systems has been developed by Chiu and his coworkers (Ju and Chiu, 1996; Ju and Chiu, 1997; Ju, et al., 1997; Ju and Chiu 1998a). Ju and Chiu (1998b) considered that these methods are still tedious and inefficient in practice owing to too many relay experiments. Thus, they proposed a modified

technique based on the fast Fourier transforms. Although the method of Ju and Chiu (1998b) has reduced a lot of relay experiments in comaprison with their previous studies, it still needs 3 relay tests for 2×2 systems, 7 relay tests for 3×3 systems, 15 relay tests for 4×4 systems, etc. Another drawback for this technique is that one needs to insert relays into the control loops one-by-one during the experimental test, and then take them off and switch back to PID (proportional-integralderivative) controllers one-by-one after the test. Besides, the calculations are cumbersome and less systematic. The published equations for the calculation of L_{c,max}, are limited to 2×2 and 3×3 systems. For a higher system, say 4×4 system, one needs to derive new equations for computation.

On the other hand, process identification with a pulse or step testing using Fourier transforms via frequency domain analysis is considered to be a well-established open-loop

technique in the process industries (ISA, 1968; Hougen, 1979). Extension of this technique to the closed-loop identification and PID controller tuning for SISO systems has been proposed by several authors (Krishnaswamy, et al., 1987; Huang, et al., 1997a; Huang, et al., 1997b). In addition. closed-loop nonparametric а identification in frequency domain using the step testing via the Fourier transforms for multivariable systems was presented by Melo and Friedly (1992). Furthermore, authors (Krishnaswamy, et al., 1987; Melo and Friedly, 1992; Huang, et al., 1997b) have also demonstrated that such approaches can tolerate some of process noise and load disturbances occurred during the closed-loop testing.

The pulse testing is a most practical and easy method obtaining relatively for experimental dynamic data in many process applications. Merits of the method include: (i) it can theoretically generate the entire frequency representation of the process, (ii) it is usually more advantageous than a step testing under the closed-loop control, since the controlled variable will ultimately return to the original value, (iii) it is the least disruptive to plant operation among the process identification methods, and (iv) it can normally get more reliable frequency response data than the step testing (Luyben, 1990, p. 519). The purpose of this study is to find the $L_{c,\max}$ under closedloop control using a pulse testing for SISO or multivariable systems. Based on the technique of Melo and Friedly (1992), the Lc.max, of a n×n multivariable system can be systematically obtained on-line by n pulse tests. Besides, the parametric identification for a given process model can also be simultaneously implemented using the same experimental data.

2. L_{C,max} for SISO systems

The L_{cmax} of a SISO system can easily be obtained by a closed-loop pulse testing. As shown in the block diagram of Fig. 1, if it is a SISO system, the process transfer function, $G_p(s)$, is assume to be unknown, and the feedback controller, $G_c(s)$, is governed by the PID mode. Under a closed-loop operation without disturbance, i.e., d(s) = 0, an arbitrary pulse in the set point, r(s), is introduced to the system. The closed-loop dynamics, $G_{ct}(s)$, can be obtained from the process response y(s) as

$$G_{CL} = \frac{y(s)}{r(s)} = \frac{G_C(s)G_P(s)}{1 + G_C(s)G_P(s)}$$
(1)

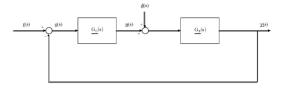


Fig. 1. The system considered.

Applying the definition of the Laplace transform to Eq. (1), and replacing s by $i\omega$ produces the Fourier integral transforms as

$$G_{CL}(i\omega) = \frac{G_{C}(i\omega)G_{P}(i\omega)}{1 + G_{C}(i\omega)G_{P}(i\omega)} = \frac{\int_{0}^{\infty} y(t)e^{-i\omega t}dt}{\int_{0}^{\infty} r(t)e^{-i\omega t}dt}$$
$$= \operatorname{Re}[G_{CL}(i\omega)] + i\operatorname{Im}[G_{CL}(i\omega)] \quad (2)$$

where Re[•] and Im[•] denote the real and imaginary parts of frequency response data, respectively. Digital evaluation of Eq. (2) from experimental pulse testing via Fourier integral transforms (FIT) has been developed by several authors (Clements and Schnelle, 1963; Messa, et al., 1969; Hougen, 1979). Listings of computer programs for data analysis are available in ISA (1968) or Luyben (1990). A parabolic approximation for the numerical FIT proposed by Messa et al. (1969), which achieves an accurate computation with fewer terms and permits multiple sampling intervals, is adopted in this study. On the other hand, the closed-loop log modulus, La, is just converted from the closed-loop magnitude ratio, $|G_{CI}(i\omega)|$, as

$$L_{C}(\omega) = 20 \cdot \log |G_{CL}(i\omega)| \tag{3}$$

The maximum closed-loop log modulus (L_{c,max},) can therefore be found by searching for the resonant peak in L_c(ω) over the entire frequency (ω) range. In addition, frequency response data for the process $G_p(s)$ can also be obtained from Eq. (2) if the transfer function or the frequency response data of the controller $G_c(s)$ is known, i.e.,

$$G_{P}(i\omega) = \frac{G_{CL}(i\omega)}{G_{C}(i\omega)(1 - G_{CL}(i\omega))}$$
(4)

3. L_{C,max} for multivariable systems

Consider a n×n multivariable system operating under PID control as shown in Fig. 1. Assume that the disturbance \underline{d} equals zero, one can have

$$\underline{y} = G_P \cdot \underline{u} \tag{5}$$

$$\underline{u} = \underline{\underline{G_C}} \cdot (\underline{r} - \underline{y}) \tag{6}$$

where y is the controlled variable vector, \underline{r} is the set-point variable vector, u is the manipulated variable vector, G_P is the process transfer function matrix, and $\boldsymbol{G}_{\!\scriptscriptstyle C}$ is the controller transfer function matrix, i.e.,

corresponding responses for each controlled variable, i.e.,
$$y_{11}(t)$$
, $y_{21}(t)$, ..., $y_{n1}(t)$, are obtained. One can use a vector $\underline{y}_1(t)$ to represent these responses. Then, this procedure is repeated for $r_2(t)$ and so on up to $r_n(t)$. Thus, n response vectors as following are obtained:

$$\begin{bmatrix} y_{11}(t) \\ y_{12}(t) \end{bmatrix}$$

Using the technique developed by Melo and Friedly (1992), if a pulse testing in the set

introduced,

is

 r_1 (t)

manipulated variable vector,
$$\underline{G_P}$$
 is the process transfer function matrix, and $\underline{G_C}$ is the controller transfer function matrix, i.e., $\underline{y}_1(t) = \begin{bmatrix} y_{11}(t) \\ y_{21}(t) \end{bmatrix}$; $\underline{y}_2(t) = \begin{bmatrix} y_{12}(t) \\ y_{22}(t) \end{bmatrix}$; $\underline{y}_2(t) = \begin{bmatrix} y_{12}(t) \\ y_{22}(t) \end{bmatrix}$; $\underline{y}_{11}(t) = \begin{bmatrix} y_{11}(t) \\ y_{21}(t) \end{bmatrix}$; $\underline{y}_{11}(t) = \begin{bmatrix} y_$

 $\underline{G_{P}} = \begin{bmatrix}
G_{P11} & G_{P12} & . & . & . & G_{P1n} \\
G_{P21} & G_{P22} & . & . & . & G_{P2n} \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
. & . & . & . & . & . \\
G_{Pn1} & G_{Pn2} & . & . & . & G_{Pnn}
\end{bmatrix} ;$

Combining Eqs. (5) and (6) and solving for \mathcal{Y} , one has

$$\underline{\underline{y}} = \left[(\underline{\underline{I}} + \underline{\underline{G}_p} \cdot \underline{\underline{G}_C})^{-1} \underline{\underline{G}_p} \cdot \underline{\underline{G}_C} \right] \cdot \underline{\underline{r}}$$
 (7)

Let r(t) represent the matrix for all set-point changes and y(t) be the matrix for the corresponding responses after these experiments,

$$y(t) = \underbrace{\begin{vmatrix} y_1(t) & y_2(t) & \dots & y_n(t) \end{vmatrix}}_{=}$$

$$= \begin{bmatrix} y_{11}(t) & y_{12}(t) & \dots & y_{1n}(t) \\ y_{21}(t) & \dots & \dots & \dots \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1}(t) & y_{n2}(t) & \dots & y_{nn}(t) \end{bmatrix}$$

Equation (7) can therefore be rewritten in the matrix form as

$$\underline{\underline{y}} = \left[\underline{\underline{I}} + \underline{\underline{G_p}} \cdot \underline{\underline{G_C}} \right]^{-1} \underline{\underline{G_p}} \cdot \underline{\underline{G_C}} \cdot \underline{\underline{r}}$$
or it can be derived as

$$\underline{\underline{G}_{P}} \cdot \underline{\underline{G}_{C}} = \underbrace{\underline{y} \cdot \underline{r}^{-1}}_{=} \cdot \left(\underline{\underline{I}} - \underline{y} \cdot \underline{\underline{r}}^{-1} \right)^{-1}$$
 (9)

The above equation can be converted into frequency domain as $G_P(i\omega)G_C(i\omega) =$

$$y(i\omega) \cdot r^{-1}(i\omega) \cdot \left[I - y(i\omega) \cdot r^{-1}(i\omega) \right]^{-1}$$
 (10)

Similar to Eq. (2), $y(i\omega)$ and $\underline{r}(i\omega)$ can be obtained from the time-domain experimental data via the Fourier integral transforms after n pulse tests. The closed-loop log modulus $L_c(\omega)$ for each frequency can therefore be calculated as (Luyben, 1990, p. 603)

$$L_{C}(\omega) = 20 \cdot \log \left| \frac{W(i\omega)}{1 + W(i\omega)} \right| \tag{11}$$

where

$$W(i\omega) = -1 + \det[\underline{\underline{I}} + \underline{\underline{G_p}}(i\omega)\underline{\underline{G_C}}(i\omega)] \quad (12)$$

Accordingly, the maximum closed-loop modulus $L_{c,\max}$ can therefore be found. In addition, if the transfer function or frequency response of each controller is known, i.e., the matrix $G_c(i\omega)$ is known, the frequency response data $G_P(i\omega)$ for each process can be derived from Eq. (10) as

$$\frac{G_{P}(i\omega)}{=} = \underbrace{y(i\omega) \cdot r^{-1}(i\omega)} \cdot \underbrace{\left[I - y(i\omega) \cdot r^{-1}(i\omega)\right]^{-1} \cdot \underline{G_{C}}^{-1}(i\omega)}_{=} = \underbrace{\left[I - y(i\omega) \cdot r^{-1}(i\omega)\right]^{-1} \cdot \underline{G_{C}}^{-1}$$

4. Parametric identification

For a SISO system, Eq. (4) has shown that the frequency response data of the process can be obtained by the pulse testing, i.e.,

$$G_p(i\omega) = \text{Re}[G_p(i\omega)] + i\text{Im}[G_p(i\omega)]$$
 (14)

is known. If these data are required to be fitted into a given parametric model $\hat{G}_n(s)$, where the superscript "A" indicates prediction or estimation of the process model, the frequency response of the model can be derived as

$$\hat{G}_{P}(i\omega) = \text{Re}[\hat{G}_{P}(i\omega)] + i \,\text{Im}[\hat{G}_{P}(i\omega)]$$
 (15)

The identification parametric can he implemented by the minimization of a squared absolute error φ in the frequency domain as

$$Min \ \Phi = Min \sum_{\omega=0}^{\omega_{\max}} \left| G_p(i\omega) - \hat{G}_p(i\omega) \right|^2$$
 (16)

Also, from the definition of the complex number, the absolute value of the difference term in Eq. (16) can be expressed as squared vector distance between data and model in the frequency domain, i.e.,

$$\left| G_p(i\omega) - \hat{G}_p(i\omega) \right| = \sqrt{(\Delta \operatorname{Re})^2 + (\Delta \operatorname{Im})^2}$$
(17)

where

$$\Delta \operatorname{Re} = \operatorname{Re} \left[G_p(i\omega) \right] - \operatorname{Re} \left[\hat{G}_p(i\omega) \right]$$
(18)

$$\Delta \operatorname{Im} = \operatorname{Im} \left[G_p(i\omega) \right] - \operatorname{Im} \left[\hat{G}_p(i\omega) \right]$$
(19)

Thus, model parameters of $\hat{G}_{p}(s)$ can therefore be obtained by a least-squares fit in the frequency domain as

$$Min \ \Phi = Min \sum_{\omega=0}^{\omega_{\max}} \left\{ \operatorname{Re}[G_{p}(i\omega)] - \operatorname{Re}[\hat{G}_{p}(i\omega)] \right\}^{2}$$

$$+ \left\{ \operatorname{Im}[G_{p}(i\omega)] - \operatorname{Im}[\hat{G}_{p}(i\omega)] \right\}^{2}$$

$$(20)$$

For a n×n multivariable system, there are n×n sets of frequency response data $G_{pij}(i\omega)$ in $\underline{G_P}(i\omega)$, and the $\underline{G_P}(i\omega)$ can be obtained by Eq. (13). Similar to the SISO system, the parameters for a given process model $\hat{G}_{pij}(s)$ can be obtained by using a minimization technique in Eq. (20). Obviously, in order to obtain all the parameters of a given multivariable model $\underline{\hat{G}_P}(s)$, one needs to run Eq. (20) using each pair of $G_{pij}(i\omega)$ and $\hat{G}_{pij}(i\omega)$ one-by-one for n×n times.

The numerical calculation for the upper frequency ω_{max} of Eq. (20) should theoretically be approximated to the infinite. However, due to the numerical errors in FIT and the limited harmonic frequency content of the input-forcing function, the computed frequency response data will normally begin to oscillate at the higher values of frequency (Luyben, 1990, p. 514). obtain meaningful results, frequency response data should be calculated only up to the frequency where it begins to oscillate. An estimate of the upper limit of frequency (ω_{max}) recommended by Clements and Schnelle (1963) was employed in this study. Clements and Schnelle (1963) defined a normalized frequency content $S(\omega)_n$ and described a comparative plot of $S(\omega)_n$ vs ωT_p for a number of pulse shapes, where T_p is the width of the input pulse. Clements and Schnelle (1963) also stated that when $S(\omega)_n$ drops to a value in neighborhood of 0.3 or 0.2, the computed frequency response would usually be unreliable. In the following simulation studies of this paper, a rectangular pulse and $S(\omega)_n < 0.3$ are adopted. Thus, according to Clements and Schnelle (1963), the upper frequency for the numerical integration in Eq. (20) can be calculated as $\omega_{\text{max}} = 5.0 / T_{\text{p}}$.

5. Examples

Example 1. Consider a SISO system, which was studied by Ju and Chiu (1998b), as

$$G_P(s) = \frac{e^{-2s}}{(s+1)(s^2+0.5s+1)}$$
 (21)

$$G_C(s) = 0.38 \left(1 + \frac{1}{1.30s} + 1.34s \right)$$
 (22)

Under the closed-loop system, a rectangular pulse (pulse height = 1.0 and pulse width = 0.5) was introduced into the set point, and the process transient y(t) with sampling time 0.05 was obtained. Figure 2 shows diagrams of the transient response y(t). Then, these data were converted into frequency domain using Eq. (2), and the parabolic FIT of Messa et al. (1969). The closed-loop log modulus $L_C(\omega)$, as shown in Fig. 3, can easily be calculated by Eq. (3), and the $L_{C,\max}$ is therefore found. The resulting $L_{C,\max}$ 3.323 dB is almost identical to

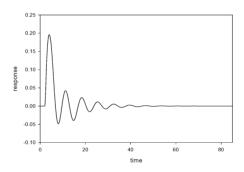


Fig 2. Pulse test response for Example 1

the actual value of 3.319 dB. Also, in comparison with 3.354 dB obtained by Ju and Chiu (1998b), it appears that the proposed method can yield a more reliable Lc,max without inserting any relay into the system. In addition, parametric identification can implemented using the same experimental data. The frequency response data $G_{\nu}(i\omega)$, shown in Fig. 4, can be obtained by using Eq. (4). It could be found from Fig. 4 that the Nyquist plot obtained from Eq. (4) is almost the same as the real process. If the process model is assumed to be the same form as Eq. (21) and the Powell (1964) algorithm is applied to find a minimization value of Φ in Eq. (20), the estimated model becomes

$$\hat{G}_{P}(s) = \frac{1.00e^{-2.00s}}{(1.00s+1)(1.00s^{2}+0.50s+1)}$$
(23)

It looks that the model parameters are almost the same as that of the original process.

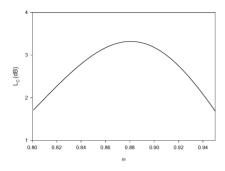


Fig. 3. Closed-loop log modulus versus frequency for Example 1.

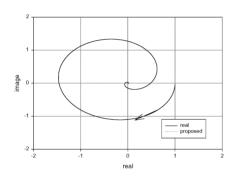


Fig. 4. Comparison of Nyquist plots of the real process and the proposed method for Example 1.

Example 2. A multivariable system, which was studied by Ogunnaike et al. (1983) and adopted by Ju and Chiu (1998b), is used to test the proposed technique, where

$$\underline{G_{p}}(s) = \begin{bmatrix} \frac{0.66e^{-2.6s}}{6.7s + 1} & \frac{-0.61e^{-3.5s}}{8.64s + 1} & \frac{-0.0049e^{-s}}{9.06s + 1} \\ \frac{1.11e^{-6.5s}}{3.25s + 1} & \frac{-2.36e^{-3s}}{5s + 1} & \frac{-0.01e^{-1.2s}}{7.09s + 1} \\ \frac{-34.68e^{-9.2s}}{8.15s + 1} & \frac{46.2e^{-9.4s}}{10.9s + 1} & \frac{0.87(11.61s + 1)e^{-s}}{(3.89s + 1)(18.8s + 1)} \end{bmatrix}$$

(24)

$$\underline{G_c}(s) = \begin{bmatrix} 1.509 \left(1 + \frac{1}{35.26s} \right) & 0 & 0 \\ 0 & -0.295 \left(1 + \frac{1}{38.7s} \right) & 0 \\ 0 & 0 & 2.629 \left(1 + \frac{1}{14.211s} \right) \end{bmatrix}$$

(25)

Under the closed-loop system, three rectangular pulses (pulse height = 1.0 and pulse width = 0.5) were introduced to each set point respectively, and each process transients with sampling time 0.05 was obtained in Fig. 5. These data were converted into frequency domain using Eq. (10), and the $L_{\rm C}(\omega)$ as shown in Fig. 6 was calculated by Eqs. (11) and (12). The resulting $L_{\rm C,max}$ obtained via the proposed method using 3 pulse tests is 4.325 dB, which is close to the actual value of 4.346 dB. In comparison with 4.435 dB obtained by Ju and Chiu (1998b) using 7 relay tests, the proposed method seems to be more versatile than theirs.

Then, the frequency response data $\underline{\underline{G_p}}(i\omega)$ were obtained by Eq. (13). Figure 7 shows the result of these frequency response data $G_p(i\omega)$

by Nyquist plots. It can be found from Fig. 7 that each of the Nyquist plots obtained by Eq. (13) is quit close to the real process. In addition, the parametric identification via the Powell (1964) algorithm to find the minimization in Eq. (20) could be implemented using the same data in frequency domain. If process models are assumed to be the same forms as the original transfer functions, i.e., Eq. (24), the fitting results are

$$\underline{\hat{G}}_{p}(s) = \begin{bmatrix} \frac{0.624e^{-2.601s}}{6.683s + 1} & \frac{-0.609e^{-3.500s}}{8.616s + 1} & \frac{-0.0049e^{-1.000s}}{9.032s + 1} \\ \frac{1.051e^{-6.500s}}{3.245 + 1} & \frac{-2.360e^{-3.000s}}{4.997s + 1} & \frac{-0.010e^{-1.200s}}{7.074s + 1} \\ \frac{-32.780e^{-9.201s}}{8.126s + 1} & \frac{46.116e^{-9.400s}}{10.874s + 1} & \frac{0.869(11.598s + 1)e^{-1.000s}}{(3.892s + 1)(18.746s + 1)} \end{bmatrix}$$

(26)

It looks that the above model parameters are quite close to that of the original process Eq. (24).

6. Conclusions

A straightforward on-line method to find the maximum closed-loop log modulus $L_{\text{c,max}}$ for SISO or multivariable systems is proposed in this study. The $L_{\text{c,max}}$ of a $n \times n$ multivariable

system can be systematically obtained on-line by n pulse tests in each of set points under closed-loop operation without adding any relay. The parametric identification can also implemented using the same set experimental data. Besides. the calculations required are easily programmed, since several for subprograms Fourier transforms optimization are available. Compared with the latest methods in the literature, the proposed method seems to be simpler and more versatile not only in the experiment but also in the numerical computation.

7. References

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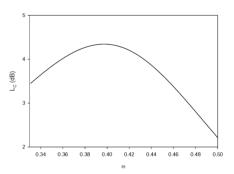


Fig. 6. Closed-loop log modulus versus frequency for Example 2

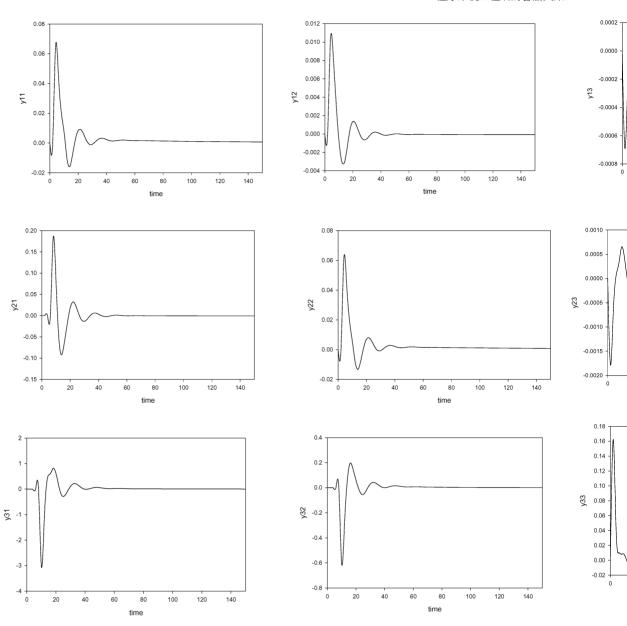


Fig. 5. Pulse test responses for Example 2.

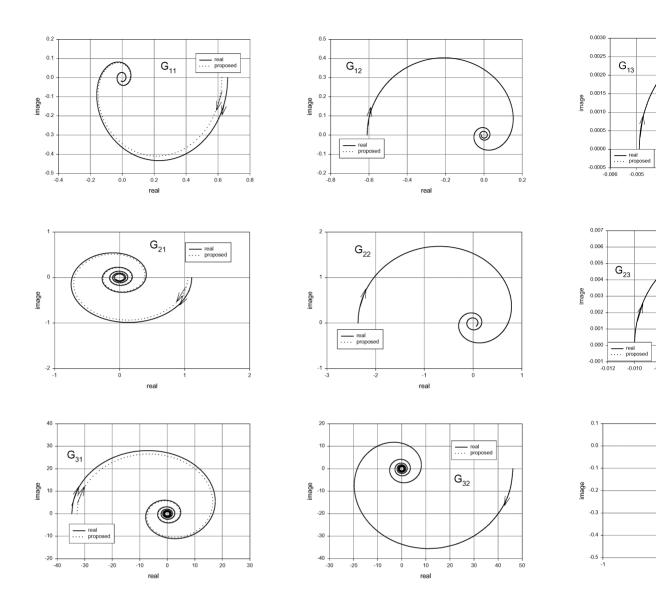


Fig. 7. Comparison of Nyquist plots of the real process and the proposed method for Example